Your Name

Your Student ID Number

Professor's Name

Lecture Section (circle one)

A (9:30), B (12:30), C (1:30), D (hybrid)

- Turn off and stow away all cell phones, watches, music players, and other electronic devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	2. $\int \frac{1}{x}\,dx = \ln\lvert x\rvert$
3. $\int e^x\,dx = e^x$	4. $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$	6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$	8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$	10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert$	12. $\int \csc x\,dx = \ln\lvert \csc x - \cot x\rvert$
13. $\int \tan x\,dx = \ln\lvert \sec x\rvert$	14. $\int \cot x\,dx = \ln\lvert \sin x\rvert$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Write your final answer in the "Answer: __________" area for each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3} + \frac{1}{7}$).
- All exam pages are double-sided except for this cover page and the last page. You may use the blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank pages at the end of the exam.
- This exam has 9 problems on 9 pages. When the exam starts, make sure that your exam is complete. Good luck!

1. (14 total points) Evaluate the following integrals. Show your work.

   (a) (7 points) $\displaystyle\int \frac{x}{\sqrt{x^2-2x-8}}\,dx$

   Answer: ______

   (b) (7 points) $\displaystyle\int \ln(1+x^2)\,dx$

   Answer: ______

2. (14 total points) Evaluate the following integrals. Show your work.

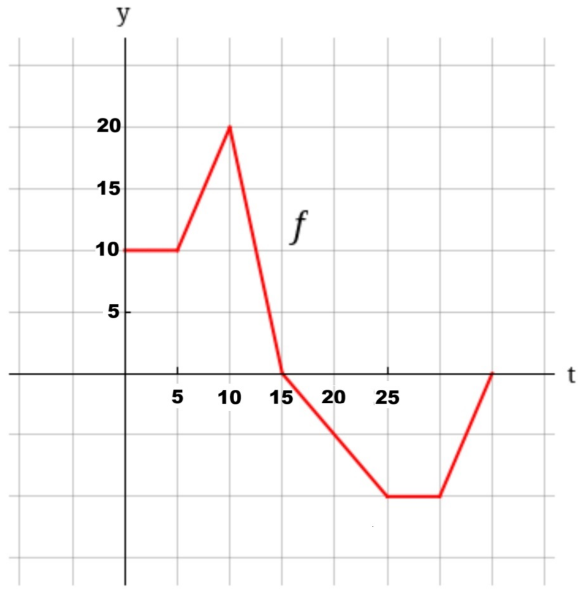
(a) (7 points) $\displaystyle\int_1^2 \frac{2}{x^3+x}\,dx$

Answer: ______________________

(b) (7 points) $\displaystyle\int_0^2 xe^{x^2} + x^{\sqrt{2}}\,dx$

Answer: ______________________

3. (10 total points) Consider the graph of the function f given below consisting of several line segments. Notice that the grid lines are in multiples of 5.



Use the graph to answer the questions below.

(a) (3 points) Find $\int_{10}^{25} f(t)\,dt$

Answer: ________________

(b) (2 points) Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{5i}{n}\right) \cdot \left(\frac{5}{n}\right)$

Answer: ________________

(c) (2 points) If $g(x) = \int_0^x f(t)\,dt$, at what value of x is $g(x)$ maximum?

Answer: At $x =$ ________________

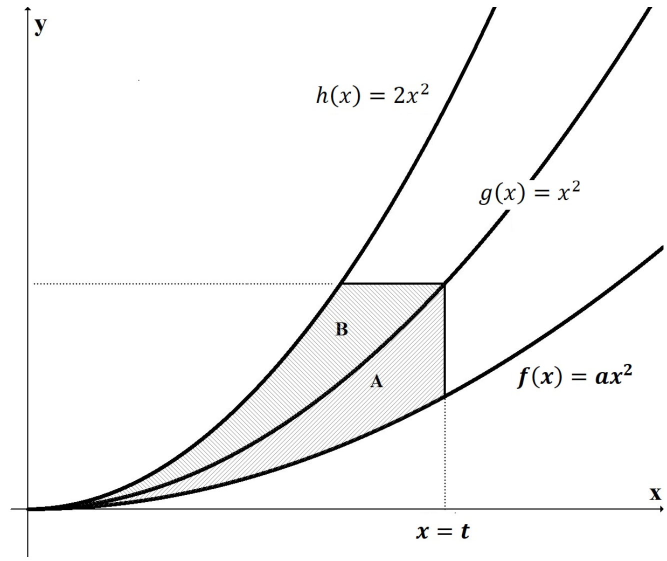
(d) (3 points) If $g(x) = \int_0^x f(t)\,dt$, then find $g''(7)$.

Answer: $g''(7) =$ ________________

4. (10 points) Let $g(x) = x^2$ and $h(x) = 2x^2$. Assume $f(x) = ax^2$, for some constant $0 < a < 1$.

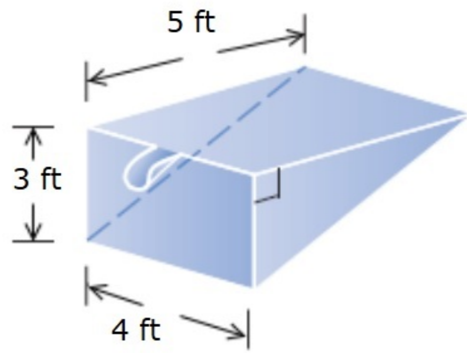
A vertical line is drawn at some positive number $x = t$ and a horizontal line is drawn through the point where $x = t$ intersects $g(x)$ (as shown). Let A and B be the regions as shown.

If the areas of A and B are the same for all positive values of t, what is the value of a?



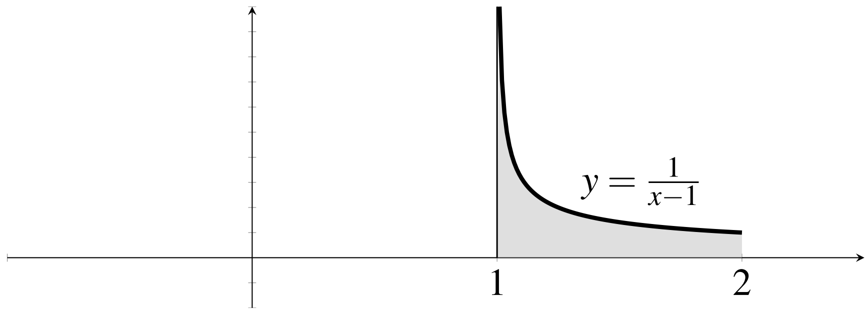
ANSWER: $a =$ ________________

5. (10 points) The tank shown below is full of water. Find the work (in ft-lb) required to pump the water out of the spout which is at the top of the tank. Use the fact that water weighs 62.5 lb/ft^3.



ANSWER: Work = ____________________ ft-lbs

6. (10 total points) Consider the region $\mathcal{R}$, bounded by the line $x = 1$, the x-axis between 1 and 2, and the curve $y = \frac{1}{x-1}$.



(a) (5 points) Does $\mathcal{R}$ have finite area? Justify your answer by computing an improper integral.

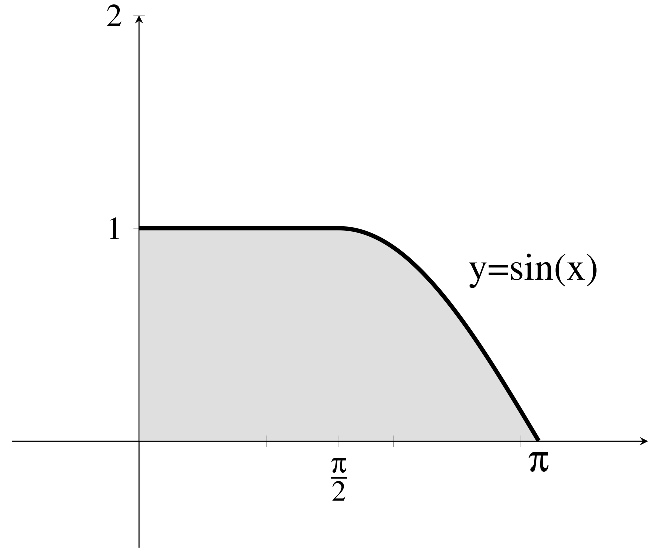
ANSWER: Area is ____________ (write "infinite" or computed value)

(b) (5 points) The solid you obtain by rotating the region $\mathcal{R}$ <u>around the line $x = 1$</u> has finite volume. Compute it.

ANSWER: Volume = __________________ cubic units

7. (10 points) Find the y-coordinate $\bar{y}$ of the centroid of the shaded region below.

   The boundary of the region consists of: the y-axis from 0 to 1, the line $y = 1$ from $x = 0$ to $x = \frac{\pi}{2}$, the curve $y = \sin x$ from $x = \frac{\pi}{2}$ to $x = \pi$, and the x axis from 0 to π.



ANSWER: $\bar{y}$ = ____________________

8. (10 points) Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{dy}{dx} = \frac{\cos^3 x}{2y}, \quad y(\pi) = -4$$

For full credit, write your answer in explicit form, $y = f(x)$.

ANSWER: $y =$ ________________________

9. (12 total points) A tank initially contains $1,000$ L of saltwater, with 100 kg of dissolved salt in it. Saltwater containing 0.5 kg/L of dissolved salt comes into the tank at a rate of 4 L/sec. The solution is kept thoroughly mixed and drains from the tank at the same rate of 4 L/sec.

   Let $y = y(t)$ be the amount of salt (in kg) in the tank after t seconds.

   (a) (3 points) Write the differential equation for $y = y(t)$ and initial condition.

   Answer: $\frac{dy}{dt} =$ ____________________ and $y(0) =$ ________

   (b) (7 points) Solve the differential equation and give $y(t)$, including finding all constants of integration.

   Answer: $y(t) =$ ____________________

   (c) (2 points) Find $\lim_{t\to\infty} y(t)$.

   Answer: $\lim_{t\to\infty} y(t) =$ ________ kg