Your Name

Student ID

Professor's Name

- Turn off and stow away all cell phones, smart watches, music players, and any other smart or connected devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). No other papers or notebooks are allowed. Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below. You must **show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \dfrac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \dfrac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
17. $\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
18. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right), \quad a > 0$

- Place a box around your answer to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\dfrac{\pi}{3}$ or $5\sqrt{3}$).
- All pages are double-sided except for this cover page and the last pages. You may use the blank pages for extra room but, if you want us to grade these spare pages, clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank page(s) at the end of the exam.
- This exam has 9 problems on 9 pages. When the exam starts, check that your exam is complete. Good luck!

1. (12 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

   (a) $\displaystyle\int \frac{x}{\sqrt{x^2+2x+10}}\,dx$

   (b) $\displaystyle\int \sin(4x)\cos(2x)\,dx$

2. (12 points) Evaluate the following integrals. Show all work. Simplify and box your answers.

(a) $\displaystyle\int_0^1 x\tan^{-1}(x)\,dx$

(b) $\displaystyle\int_0^\infty \frac{1}{2x^2+7x+3}\,dx$

3. (10 points) The velocity of a particle is given by

$$v(t) = 9(t-1)e^{3t} \quad \text{in feet/sec, where } t \text{ is in seconds}$$

(a) Assume the initial position of the particle is $s(0) = 0$.
Find the function $s(t)$ for the position of the particle at time t.

(b) Find the **total distance** traveled by the particle from $t = 0$ to $t = 2$ seconds.

4. (10 points) Find the arc length of the curve

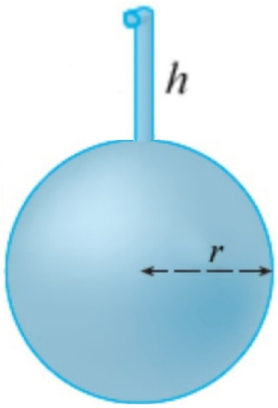
$$y = \frac{2}{3}(x^2+1)^{3/2}$$

from $x = -1$ to $x = 1$.

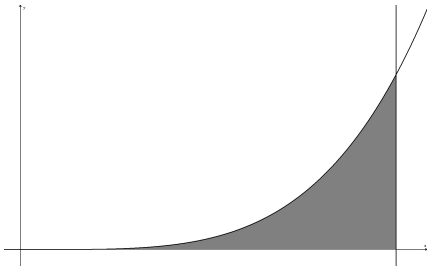
5. (10 points) The spherical tank shown has a radius of $r = 1$ ft and a spout at the top of tank of height $h = 1$ ft.

   The tank is *half* full of oil that has a density of 60 lbs/ft^3.

   Find the work required to pump the oil out of the spout.



6. (12 points) Consider the region R bounded by the curve $y = x^4$, the line $x = 2$, and the x-axis.



(a) Find the value of a such that the line $x = a$ would divide the region R into two regions of equal area.

(b) A solid is obtained by rotating the region R around the horizontal line $y = -3$.
SET UP two integrals which are equal to the volume of this solid, the first integral by applying the cylindrical shells method, and the second one by the washers methods. (DO NOT EVALUATE)

SHELLS:

WASHERS:

7. (12 points) Find the center of mass of the quarter-ellipse region in the first quadrant bounded by

$$y = \sqrt{1 - \left(\frac{x}{2}\right)^2}, \text{ the } x\text{-axis, and the } y\text{-axis.}$$

8. (10 points) Find the solution of the following initial value problem.

$$y' = (y^2 - 3y + 2)\cos(x), \qquad y(0) = 3.$$

For full credit, write your answer in explicit form, $y = f(x)$.

9. (12 points) A radioactive material decays at a rate proportional to the amount present (so if $y(t)$ is the amount, $y'(t) = -\alpha y(t)$ for some positive number α.)

   (a) On day 0 there are 10 kg of material present. On day 3, there are 8 kg of material present. After how many days will there only be 2 kg?

   (b) Suppose the scientists start on day 0 with 12 kg of the same material, and remove 0.5 kg every day, so the new differential equation is $y' = -\alpha y - 0.5$ (with the same α as before.) After how many days will there be only 2 kg left?

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