Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	14	
2	14	
3	13	
4	14	
5	10	

Question	Points	Score
6	10	
7	13	
8	12	
Total	100	

1. (14 points)

   (a) (7 points) Evaluate the integral $\int \frac{1}{x^3 - 4x^2}\,dx$. Show your work, and box your answer.

   (b) (7 points) Evaluate the following improper integral, if it converges, or show why it diverges.

$$\int_0^{\infty} \frac{e^x}{1+e^{2x}}\,dx$$

2. (14 points)

   (a) (7 points) Evaluate $\int_0^{\sqrt{3}} x\tan^{-1}(x)dx$.
   Give your answer in exact form (in terms of square roots and/or multiples of π).

   (b) (7 points) Find the function $f(x)$ if $f'(x) = \dfrac{1}{(r^2 - x^2)^{3/2}}$ and $f(0) = 0$.
   The constant r should appear in your answer.

3. (13 points) The velocity of a particle is given by $v(t) = \sin^3(\pi t)$ ft/sec where t is in seconds.

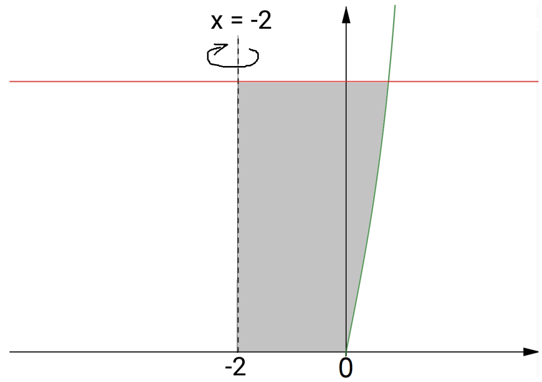
   (a) (7 points) Assume the initial position of the particle is $s(0) = 0$ ft.
   Find the function $s(t)$ for the position of the particle at time t.

   (b) (6 points) Find the total distance traveled by the particle from $t = 0$ to $t = \frac{3}{2}$ seconds.

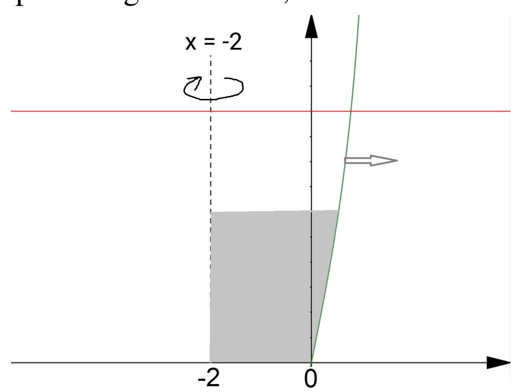
4. (14 points) Let R be the region enclosed by: the x-axis, the line $y = 5$, the line $x = -2$, and the portion of the curve $y = 5\tan(x)$ between $x = 0$ and $x = \pi/4$. The region R is rotated around the line $x = -2$ to form a solid of revolution. The units are meters. In parts (b) and (c) take g to be 9.8 m/sec^2 and take the density of water to be 1000 kg/m^2.

   Write each of the following in terms of integrals, but do **not** evaluate the integrals.

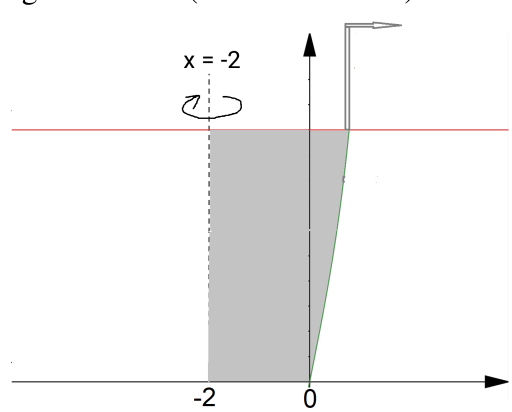
   (a) (7 points) the volume of the resulting container;



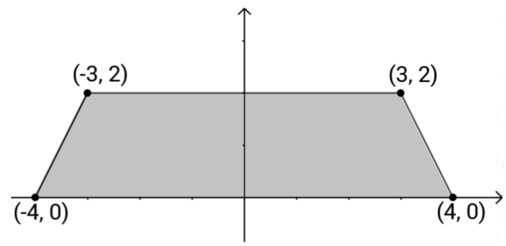
   (b) (4 points) the amount of work (in Joules) required to empty the container of water, if water is filled up to the level of 3 meters, and there's an outtake pipe at height 4 meters;



   (c) (3 points) the amount of work (in Joules) required to empty the container of water if the container is filled to the top with water and the outtake pipe is at height 7 meters (above the x-axis).



5. (10 points) Find the coordinates $(\bar{x}, \bar{y})$ for the center of mass of the region shown below.



6. (10 points) Find the explicit solution $y = y(x)$ to the initial value problem

$$\frac{dy}{dx} = y^2 e^{\sqrt{x}}\ ,\ y(0) = \frac{1}{5}.$$

7. (13 points) Suppose you **drop** a stone of mass m from a great height in the earth's atmosphere, and the only forces acting on the stone are the earth's gravitational attraction and a retarding force due to air resistance, which is proportional to the velocity v. Take downward to be the positive direction. Then, since $F = ma$ and $a = dv/dt$, we have the differential equation:

$$m\frac{dv}{dt} = mg - kv,$$

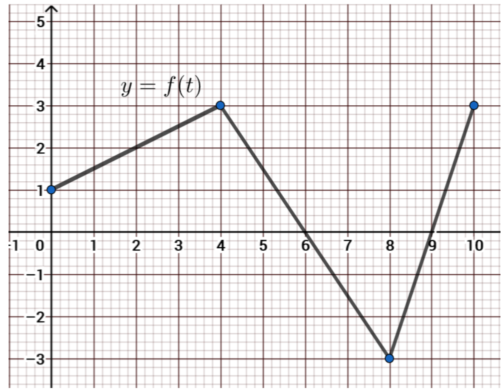
where k is a positive constant. Suppose that the mass is $m = 1$ kg, and take $g = 9.8$ m/sec^2.

(a) (6 points) Solve the differential equation to find a formula for $v(t)$. Your answer will involve k.

(b) (3 points) Compute the terminal velocity v_∞ (the limiting velocity as $t \to \infty$). Your answer will involve the positive constant k.

(c) (4 points) If $v_\infty = 70$ m/sec, find the speed of the stone after 3 sec.

8. (12 points) Suppose that the graph of f is as shown:



(a) (4 points) Compute the average value of this function over the interval $[0,10]$.

(b) Define a new function $A(x) = \int_{x}^{x^3} f(t)\,dt$, where f is the same function as above.

i. (2 points) Compute $A(2)$.

ii. (6 points) Compute $A'(2)$.