• Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.

• This exam is closed book. You may use one 8.5'' × 11'' sheet of handwritten notes (both sides OK). Do not share notes.

• You can use only a Texas Instruments TI-30X IIS calculator.

• In order to receive credit, you must show all of your work. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.

• You may use any of the 20 integrals in the table on p. 495 of the text without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.

• Place a box around your answer to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example \( \frac{\pi}{3} \) or \( 5\sqrt{3} \)).

• If you need more room, use the backs of the pages and indicate that you have done so.

• This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

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Total 100
1. (14 points)
   (a) (7 points) Evaluate the integral \( \int \frac{1}{x^3 - 4x^2} \, dx \). Show your work, and box your answer.

   (b) (7 points) Evaluate the following improper integral, if it converges, or show why it diverges.

   \[ \int_{0}^{\infty} \frac{e^x}{1 + e^{2x}} \, dx \]
2. (14 points)

(a) (7 points) Evaluate $\int_0^{\sqrt{3}} x\tan^{-1}(x)\,dx$.
Give your answer in exact form (in terms of square roots and/or multiples of $\pi$).

(b) (7 points) Find the function $f(x)$ if $f'(x) = \frac{1}{(r^2 - x^2)^{3/2}}$ and $f(0) = 0$.
The constant $r$ should appear in your answer.
3. (13 points) The velocity of a particle is given by \( v(t) = \sin^3(\pi t) \) ft/sec where \( t \) is in seconds.

(a) (7 points) Assume the initial position of the particle is \( s(0) = 0 \) ft. Find the function \( s(t) \) for the position of the particle at time \( t \).

(b) (6 points) Find the total distance traveled by the particle from \( t = 0 \) to \( t = \frac{3}{2} \) seconds.
4. (14 points) Let $R$ be the region enclosed by: the $x$-axis, the line $y = 5$, the line $x = -2$, and the portion of the curve $y = 5 \tan(x)$ between $x = 0$ and $x = \pi/4$. The region $R$ is rotated around the line $x = -2$ to form a solid of revolution. The units are meters. In parts (b) and (c) take $g$ to be 9.8 m/sec$^2$ and take the density of water to be 1000 kg/m$^2$.

Write each of the following in terms of integrals, but do not evaluate the integrals.

(a) (7 points) the volume of the resulting container;

(b) (4 points) the amount of work (in Joules) required to empty the container of water, if water is filled up to the level of 3 meters, and there’s an outtake pipe at height 4 meters;

(c) (3 points) the amount of work (in Joules) required to empty the container of water if the container is filled to the top with water and the outtake pipe is at height 7 meters (above the $x$-axis).
5. (10 points) Find the coordinates \((\bar{x}, \bar{y})\) for the center of mass of the region shown below.
6. (10 points) Find the explicit solution $y = y(x)$ to the initial value problem

$$\frac{dy}{dx} = y^2 e^{\sqrt{x}}, \ y(0) = \frac{1}{5}.$$
7. (13 points) Suppose you drop a stone of mass \( m \) from a great height in the earth’s atmosphere, and the only forces acting on the stone are the earth’s gravitational attraction and a retarding force due to air resistance, which is proportional to the velocity \( v \). Take downward to be the positive direction. Then, since \( F = ma \) and \( a = dv/dt \), we have the differential equation:

\[
m\frac{dv}{dt} = mg - kv,
\]

where \( k \) is a positive constant. Suppose that the mass is \( m = 1 \) kg, and take \( g = 9.8 \) m/sec\(^2\).

(a) (6 points) Solve the differential equation to find a formula for \( v(t) \). Your answer will involve \( k \).

(b) (3 points) Compute the terminal velocity \( v_\infty \) (the limiting velocity as \( t \to \infty \)). Your answer will involve the positive constant \( k \).

(c) (4 points) If \( v_\infty = 70 \) m/sec, find the speed of the stone after 3 sec.
8. (12 points) Suppose that the graph of $f$ is as shown:

(a) (4 points) Compute the average value of this function over the interval $[0, 10]$.

(b) Define a new function $A(x) = \int_{x}^{x^3} f(t) \, dt$, where $f$ is the same function as above.

i. (2 points) Compute $A(2)$.

ii. (6 points) Compute $A'(2)$. 