Math 125 **Final Examination** **Autumn 2015**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can use a Texas Instruments TI-30X IIS calculator. No other calculators are permitted.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Evaluate the following definite integrals.

   (a) (5 points) $\displaystyle\int_0^4 \frac{6x+5}{2x+1}\,dx$

   (b) (5 points) $\displaystyle\int_1^3 \frac{dx}{x^{3/2}+x^{1/2}}$

2. Evaluate the following indefinite integrals.

   (a) (5 points) $\displaystyle\int \frac{dx}{x(x^2-1)^{3/2}}$ (Here we assume $x > 1$.)

   (b) (5 points) $\displaystyle\int \frac{1}{x^2}\tan^{-1}(x)\,dx$

3. Let R be the (infinite) region under the graph of the function $f(x) = x^{-3/2}$, above the x-axis, and to the right of the line $x = 1$.

   (a) (5 points) Determine whether the area of the region R is finite. If it is finite, find its value and give the answer as an exact number. You must justify your answer.

   (b) (5 points) A solid of revolution is formed by rotating the region R about the y-axis. Determine whether the volume of this solid of revolution is finite. If it is finite, find its value and give the answer as an exact number. You must justify your answer.

4. (10 points) A leaky bucket was pulled from the ground up to 8 meters above the ground at a constant rate of 0.5 meters per second. Initially, the bucket contained 25 liters of water. The water was leaking out of the bucket at a constant rate of 0.3 liters per second.

   Assume that both the bucket and the rope have neglible weight. The acceleration due to gravity is 9.8 m/s^2. The density of water is 1 kilogram per liter.

   Find the total amount of work done (in joules) when the bucket of water was moved from the ground to 8 meters above the ground.

5. (10 points) Find the center of mass of the region below the graph of $f(x) = \sin^2 x$, above the x-axis, between $x = 0$ and $x = \pi/2$. Give your answer in exact form.

6. (a) (4 points) Find the area under the curve $y = 1/x^3$ and above the x-axis for $1 \leq x \leq 2$.

(b) (6 points) Find the number a such the line $x = a$ divides the region in part (a) into two parts of equal area.

7. Let $\mathscr{R}$ be the region bounded by the curve $y = \sqrt{x}$, the line $x = 4$, and the x-axis.

   (a) (6 points) The region $\mathscr{R}$ is rotated around the line $x = 6$ to form a solid. Set up an integral for the volume of this solid using cylindrical shells and EVALUATE THE INTEGRAL.

   (b) (4 points) Set up an integral for the volume of this solid using washers. DO NOT EVALUATE THE INTEGRAL.

8. Consider the curve $y = \sin x$.

   (a) (4 points) Set up a definite integral for the arc length of this curve for $0 \leq x \leq \pi/2$.
   DO NOT EVALUATE THE INTEGRAL.

   (b) (6 points) Use the Trapezoid Rule with $n = 3$ subintervals to estimate the integral in part (a). Give your answer either in exact form, or in decimal form correct to at least the third digit after the decimal point.

9. (10 points) Find the solution of the differential equation

$$y' = 1 + x + y^2 + xy^2$$

that satisfies the initial condition $y(1) = 0$.

10. A frozen pie at temperature $-10°$C is removed from a freezer and put into a refrigerator to defrost. The refrigerator is kept at a constant temperature of $4°$C. After 4 hours, the temperature of the pie is $-6°$C.

    Recall that Newton's Law of Cooling states that the rate of change of the temperature of the pie is proportional to the temperature difference between the pie and its surroundings.

    (a) (5 points) Find the temperature $T(t)$ in °C of the pie as a function of the time t in hours after it was put into the refrigerator.

    (b) (5 points) How much longer (*after* the first 4 hours) do you need to wait until the pie is at a temperature of $2°$C? Express your answer in hours, to the nearest hundredth of an hour.