- This exam is closed book. You may use one 8.5” × 11” sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.

- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).

- **No calculators** of any kind are allowed.

- In order to receive credit, you must show all of your work. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.

- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet.**

- Place a box around your answer to each question.

- If you need more room, use the backs of the pages and indicate that you have done so.

- Raise your hand if you have a question.

- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

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Total 100
1. (10 total points) Evaluate the following definite integrals.

(a) (5 points) \( \int_{0}^{\pi/10} \sin^2(5x) \cos^3(5x) \, dx \)

(b) (5 points) \( \int_{1}^{3} \frac{x^2}{x^2 - 2x + 5} \, dx \)
2. (10 total points) Evaluate the following indefinite integrals.

(a) (5 points) \[ \int \frac{dx}{x^2 \sqrt{4 + x^2}} \]

(b) (5 points) \[ \int \frac{x^{1/4} + 1}{x(x^{1/4} - 2)^2} \, dx \]
3. (10 points) Determine whether the following improper integral converges and, if so, evaluate it. Justify your answer.

\[ \int_{1}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} \, dx \]
4. (10 points) The position $s(t)$ of an object moving along a straight line is given by the formula

$$s(t) = \int_{\pi}^{t} e^{\sin x} \, dx.$$  

Find a formula for the acceleration $a(t)$ of the object as a function of time $t$. 
5. (10 points) In the graph below, the line is tangent to the curve $y = x^3 - 3x^2 - 6x + 8$ at the point on the y-axis. Find the area enclosed between the curve and the line as shown in the figure.
6. (10 points) A manufacturer drills a round hole of radius $r$ through the center of a metal sphere of radius $R$. Find the volume of the remaining metal “bead.”
7. (10 points)

A tank (shown in the figure on the left) is 16 feet high, with an open rectangular top of width 3 feet and length 4 feet. Each horizontal cross-section of the tank is a rectangle of fixed width 3 feet and length that changes with height. The figure on the right shows the front face of the tank, which has the shape of the function $y = x^4$ for $-2 \leq x \leq 2$.

Initially, there is fluid in the tank up to a height of 1 foot. The fluid weighs 15 lb/ft$^3$. How much work does it take to empty the tank by pumping all of the fluid to the top of the tank?
8. (10 total points)

(a) (4 points) Write a definite integral for the arclength $L$ of the graph of $y = x^2$ from $x = 1$ to $x = 3$. DO NOT EVALUATE THE INTEGRAL.

(b) (6 points) Use Simpson’s Rule with $n = 4$ subintervals to approximate the definite integral in part (a). Give your answer in exact form.
9. (10 points) Find the solution of the initial value problem

\[ \frac{dy}{dt} = t \sin(t) \cos^2(y), \quad y(0) = \frac{\pi}{4}. \]

Give your answer in the form \( y = f(t) \).
10. (10 total points) An advertising company introduces a new product to Seattle. Let \( P = P(t) \) be the number of people in thousands who are aware of this new product at time \( t \) in days. Seattle has a total population of 700 thousand. Initially, no one has heard of the product.

(a) (3 points) Set up a differential equation for \( P \) if it increases at a rate proportional to the number of people in Seattle still unaware of the product. Also state the initial condition for \( P \).

(b) (5 points) After 20 days, 140 thousand people have heard of the product. Solve the differential equation for \( P(t) \), and also determine the exact value of any constants in your solution.

(c) (2 points) How long does it take before half the population has heard of the product? Give your answer in exact form.