Math 125 **Final Examination** **Autumn 2013**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is needed, but no calculator with graphing, programming, symbolic manipulation, or calculus capabilities is allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 total points) Evaluate the following indefinite integrals.

   (a) (5 points) $\int x^2 \ln x \, dx$

   (b) (5 points) $\int \tan^3(x) \sec(x) \, dx$

2. (10 total points) Evaluate the following definite integrals.

   (a) (5 points) $\displaystyle\int_0^{\ln 2} \frac{7e^{2t}}{e^{2t}+3e^t+2}\,dt$

   (b) (5 points) $\displaystyle\int_0^{1/2} \sqrt{2x-x^2}\,dx$

3. (10 total points) The velocity function (in meters per second) for a particle moving along a line is given by $v(t) = t^2 - 5t + 6$.

   (a) (5 points) Find the displacement of the particle during the time interval $0 \leq t \leq 4$.

   (b) (5 points) Find the total distance traveled by particle during the time interval $0 \leq t \leq 4$.

4. (10 total points)

Consider the shaded region in the figure, bounded (in clockwise order from the origin, as shown in the figure) by

the y-axis,

the line $y = 1$,

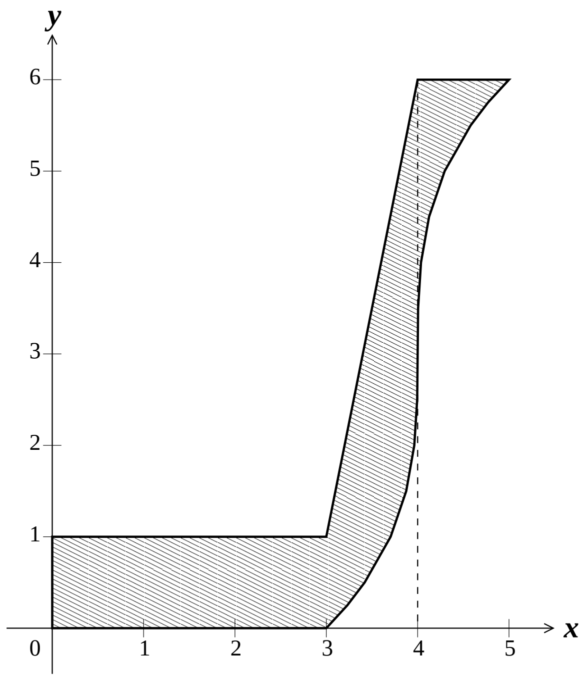
the line $y = 5x - 14$,

the line $y = 6$,

the curve $y = 3 + 3(x-4)^{1/3}$, and

the x-axis.

A Hallowe'en trick-or-treat bucket is formed by rotating this region around the y-axis.



(a) (5 points) Set up an integral (or a sum of integrals) using *SHELLS* which equals the volume of plastic needed to construct the bucket. DO NOT EVALUATE THE INTEGRAL(S).

(b) (5 points) Set up an integral (or a sum of integrals) using *WASHERS* which equals the volume of plastic needed to construct the bucket. DO NOT EVALUATE THE INTEGRAL(S).

5. (10 points) Find the coordinates of the center of mass of a circular plate of radius 1 with center at the origin $(0,0)$ made with a material whose density is 2 on the upper semicircular region and 1 on the lower semicircular region.

6. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dx} = xe^{2x^2}(1+y^2), \quad y(0) = \sqrt{3}.$$

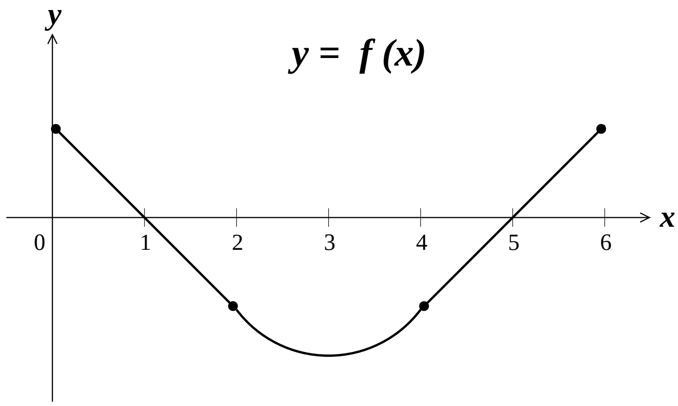
Give your answer in the form $y = f(x)$.

7. (10 total points) The stale air in a crowded exam room initially contains 6.25 ft^3 of carbon dioxide (CO_2). An air conditioner is turned on at time $t = 0$ and blows fresher air into the room at a rate of 500 ft^3/min. The fresher air mixes with the stale air (assume it mixes instantaneously) and the well-mixed air leaves the room at the same rate of 500 ft^3/min. The incoming fresher air contains 0.01% CO_2 (by volume), and the air in the room has a total volume of 2500 ft^3. By their breathing, the people in the room generate an additional 0.08 ft^3 of CO_2 per minute (without changing the total volume of air in the room). Let $y(t)$ denote the amount (in ft^3) of CO_2 in the room, t minutes after the air conditioner is turned on.

   (a) (4 points) Find a differential equation satisfied by $y(t)$. Simplify the differential equation, but wait until part (b) to solve it.

   (b) (6 points) Now solve the differential equation from part (a), and solve for any constant(s) in your solution to find a formula for $y(t)$.

8. (10 total points) The graph of $y = f(x)$ is given by



Let $A(x) = \int_0^x f(t)\,dt$ for $0 \le x \le 6$.

(a) (2 points) On what interval(s) in x is $A(x)$ positive?

(b) (2 points) On what interval(s) in x is $A(x)$ increasing?

(c) (2 points) On what interval(s) in x is the graph of $y = A(x)$ concave up?

(d) (2 points) At what value(s) of x does $A(x)$ have an absolute maximum?

(e) (2 points) On what interval(s) in x is the graph of $y = (A(x))^2$ increasing?

9. (10 points) Find the slope m of the line $y = mx$ through the origin that divides the region bounded between the parabola $y = 2x - x^2$ and the x-axis into two regions with equal area.

10. (10 total points)

(a) (5 points) In this part, treat b and c as constants. Evaluate the definite integral

$$\int_0^b \left(\frac{2x}{x^2+1} - \frac{c}{5x+1} \right) dx.$$

Your answer will involve both b and c.

(b) (5 points) Use your answer to part (a) to find the value of the constant c for which the improper integral

$$\int_0^\infty \left(\frac{2x}{x^2+1} - \frac{c}{5x+1} \right) dx$$

converges, and evaluate the improper integral for this value of c.