Math 125 **Final Examination** **Autumn 2010**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is allowed, but graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 484 of the text (p. 506 if you have the 5th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	10	
4	10	
5	8	

Question	Points	Score
6	10	
7	8	
8	10	
9	10	
10	10	
Total	100	

1. (12 total points) Evaluate the following definite integrals.

   (a) (6 points) $\int_0^{\pi/6} (\sin 2x)(\cos 2x)\, dx$

   (b) (6 points) $\int_2^3 \frac{8x^3}{2x^2 - x - 1}\, dx$

2. (12 total points) Evaluate the following indefinite integrals.

   (a) (6 points) $\int x(7-x)^{2010}\,dx$

   (b) (6 points) $\int x^3 \sin\left(x^2+1\right)\,dx$

3. (10 total points) Let $\mathscr{R}$ be the region bounded by the curve $y = x^4$, the line $x = 2$, and the x-axis.

   (a) (2 points) Sketch the region $\mathscr{R}$.

   (b) (4 points) The region $\mathscr{R}$ is rotated around the line $x = 3$ to form a solid. Set up an integral for the volume of this solid using *CYLINDRICAL SHELLS* and *EVALUATE THE INTEGRAL*.

   (c) (4 points) Set up an integral for the volume of this solid using *WASHERS*.
   DO NOT EVALUATE THE INTEGRAL.

4. (10 points) Find the area of the region enclosed between the curve $y = \dfrac{1}{(x^2 - 8x + 25)^{3/2}}$ and the line $y = \dfrac{1}{125}$. (Hint: To solve the equation to find the intersections, raise both sides to the $2/3$ power.)

5. (8 total points) Water is drawn from a well that is 35 meters deep using a leaky bucket that initially scoops up 20 kilograms of water from the bottom of the well. The mass of the bucket itself is 2 kilograms and the mass of the rope that is attached to the bucket is 0.2 kg/m. The rope is being pulled at a constant rate of 0.5 m/s. The bucket has a hole in it and water leaks from the bucket at a rate of 0.1 kg/s.

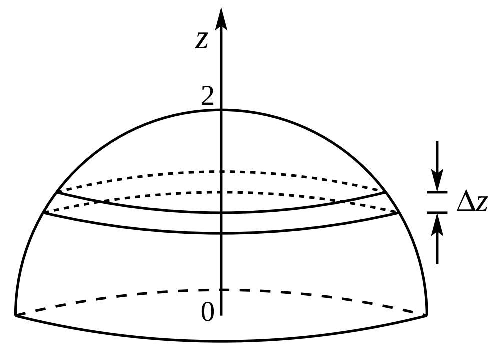
   (a) (3 points) Let y be the height (in meters) of the bucket above the bottom of the well. What is the mass of the water in the bucket when the bucket is y meters high?

   (b) (5 points) The acceleration due to gravity is 9.8 m/s^2. Find the work done when the bucket is pulled from the bottom to the top of the well.

6. (10 total points)
A mound of snow is in the shape of a hemisphere of radius 2 feet. The snow is denser at the bottom than at the top because it compresses. Suppose the density of the snow is

$$\rho(z) = \frac{15}{z^2+9}$$

pounds per cubic foot at height z feet above the bottom of the mound.



In this problem, you will set up a definite integral for the weight of the mound of snow. Imagine dividing the mound into n thin horizontal slices of equal thickness Δz.

(a) (3 points) Each slice can be approximated by a disk. Find the (approximate) volume of a typical slice (the i^{th} slice) in terms of its height z_i above the bottom of the mound and Δz.

(b) (3 points) Find the (approximate) weight of this typical slice.

(c) (2 points) Write a sum which approximates the weight of the mound of snow.

(d) (2 points) For what definite integral is this a Riemann sum?
DO NOT EVALUATE THE INTEGRAL.

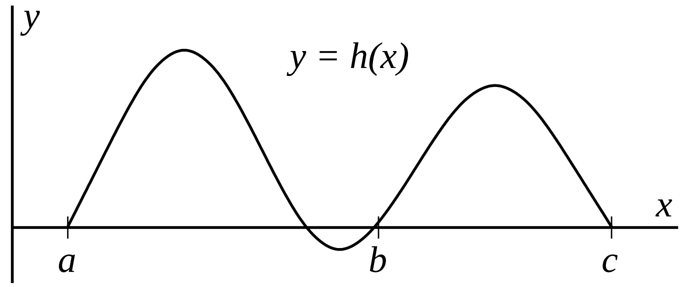
7. (8 points) Consider the improper integral $\int_2^\infty \frac{\ln x}{x^4}\,dx$.

   Evaluate the integral (if it converges) or explain carefully why it does not converge.

8. (10 points) Find the solution of the differential equation $\dfrac{dL}{dt} = 3L^2\sqrt{100 - t^2}$ that satisfies the initial condition $L(0) = -\dfrac{1}{2}$.

9. (10 points) A tank initially contains 1000 L of pure water. Brine that contains 0.07 kg of salt per liter of water enters the tank at a rate of 5 L/min. In addition, brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. Find the amount of salt (in kg) in the tank as a function of time t in minutes.

10. (10 total points) Consider the graph of $y = h(x)$:



(a) (2 points) Is $\int_a^b h(x)\,dx$ positive or negative?

Circle one: Positive Negative

(b) (2 points) Is $\int_c^a h(x)\,dx$ positive or negative?

Circle one: Positive Negative

(c) (2 points) Let $f(x)$ be a function whose derivative $f'(x)$ is defined and continuous for all real numbers x. Is it always true that for any real number a,

$$f(a) = f(0) + \int_0^a f'(x)\,dx?$$

Circle one: Always true Might be false

(d) (2 points) Let $f(x)$ be a function that is continuous and increasing for all real numbers x, and let

$$g(x) = \int_0^x f(t)dt.$$

Is it always true that $g(x)$ is increasing for all real numbers x?

Circle one: Always true Might be false

(e) (2 points) Let a be a real number. Is it always true that

$$\int_0^a \sin x\,dx \leq \int_0^a |\sin x|\,dx?$$

Circle one: Always true Might be false