1. Stewart, section 7.1: #1, 5, 7, 15, 19, 27, 33, 35, 62, 63, 65

2. Stewart, section 7.2: #5, 9, 13, 19, 27, 57, 65

3. Stewart, section 7.3: #3, 5, 7, 25, 26, 27, 28, 33, 42

4. A doughnut has been partially eaten by a meticulous person so that the portion remaining is given by rotating the half circular region shown about the y-axis. What proportion of the doughnut remains, assuming that the doughnut was the volume of revolution of the full circle?

5. A pair of functions \( f \) and \( g \) are said to be orthogonal on the interval \([a, b]\) if \( \int_a^b f(x)g(x) \, dx = 0 \). A family of functions is said to be orthogonal on \([a, b]\) if each distinct pair of functions \( f \neq g \) in the family is orthogonal on \([a, b]\).

Show that the family of trigonometric functions

\[ \{ \sin(x), \cos(x), \sin(2x), \cos(2x), \ldots, \sin(nx), \cos(nx), \ldots \} \]

is orthogonal on the interval \([0, 2\pi]\). (You need to show that the pairs \( \sin(Mx), \sin(Nx) \) and \( \cos(Mx), \cos(Nx) \) are orthogonal for all positive integers \( M \neq N \), and that the pair \( \sin(Mx), \cos(Nx) \) is orthogonal for all positive integers \( M \) and \( N \).)

This result about the orthogonality of the sine and cosine functions is the basis for an important area of mathematics called Fourier series.

6. a) Determine the average values of \( x, x^2, x^3, \) and \( x^4 \) on the interval \([1, 2]\).

b) Determine the average values of \( \sin(x), \sin^2(x), \sin^3(x), \) and \( \sin^4(x) \) on the interval \([0, \pi]\).

c) Explain geometrically why the average values in a) increased, while those in b) decreased.