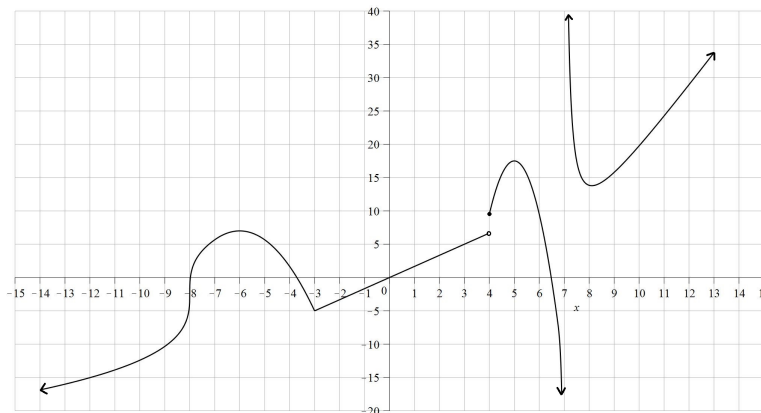


Worksheet for Week 4: Limits and Derivatives

This worksheet reviews limits and the definition of the derivative with graphs and computations.

1. Answer the following questions using the graph $y = f(x)$ below. The function $f(x)$ has domain all numbers except 7 as seen from the graph.



(a) $\lim_{x \rightarrow 4} f(x) =$

(b) $\lim_{x \rightarrow 7^+} f(x) =$

(c) $f'(0) =$

(d) $\lim_{x \rightarrow -3} f(x) =$

(e) $\lim_{x \rightarrow 0} \frac{f(x)}{x} =$

(f) $\lim_{h \rightarrow 0} \frac{f(3+h) - 5}{h} =$

(g) $f'(5) =$

(h) $\lim_{h \rightarrow 0^+} \frac{f(-8+h) - f(-8)}{h} =$

(i) $\lim_{h \rightarrow 0} \frac{f(-8+h)}{h} =$

(j) $\lim_{h \rightarrow 0} \frac{f(-6+h) - f(-6)}{h} =$

(k) $\lim_{h \rightarrow 0^+} \frac{f(-3+h) + 5}{h} =$

- (l) List all the intervals where the derivative $f'(x)$ is negative.

- (m) List all the intervals where the derivative $f'(x)$ is decreasing.

- (n) A critical value for $f(x)$ is any x in the domain of $f(x)$ where $f'(x) = 0$ or $f'(x)$ is undefined. List all critical values of $f(x)$.

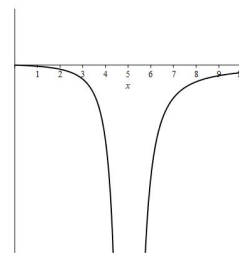
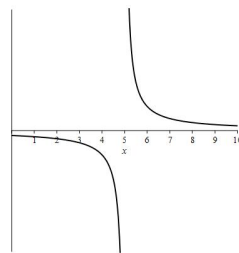
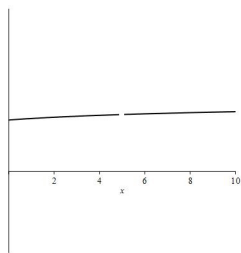
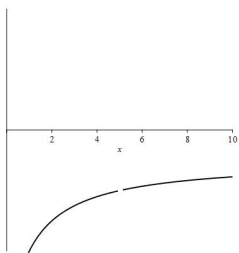
2. Evaluate the following limits and then match the functions with their graphs shown below using your limit results. Some will require you to compute left and right hand limits.

$$(a) \lim_{x \rightarrow 5} \frac{1}{x-5} =$$

$$(b) \lim_{x \rightarrow 5} \frac{-x}{(x-5)^2} =$$

$$(c) \lim_{x \rightarrow 5} \frac{-x^2 - 2x + 35}{x^2 - 4x - 5} =$$

$$(d) \lim_{x \rightarrow 5} \frac{x - \sqrt{3x+10}}{x-5} =$$

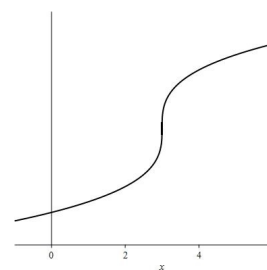
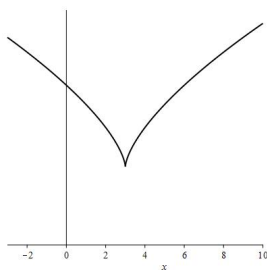
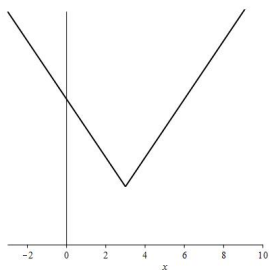


3. Use the definition of the derivative $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ to compute $f'(3)$ for the following functions. Then match the functions with their graphs shown below using your limit results.

(a) $f(x) = (x - 3)^{\frac{1}{3}} + 2$

(b) $f(x) = (x - 3)^{\frac{2}{3}} + 2$

(c) $f(x) = |x - 3| + 2$



4. Find, if any, the horizontal asymptotes of the following functions and use that information to match them with their graphs on the next page. Each question should have two limit computations with $x \rightarrow \infty$ and $x \rightarrow -\infty$.

(a) $f(x) = \frac{(x+1)^4}{x^4 + 3x^2 + 7x + 10}$

(b) $f(x) = \frac{x+3}{x^2 + 8x + 26}$

(c) $f(x) = \frac{x^3 + 4x + 9}{x^2 + 4}$

$$(d) f(x) = -7x^4 + x^3 - 12x + 20$$

$$(e) f(x) = \frac{\sqrt{8x^2 + 4}}{x + 2}$$

$$(f) f(x) = 3e^x$$

$$(g) f(x) = 7 - e^{-x}$$

When you match the functions with these graphs, add (if any) horizontal asymptotes to the pictures.

