Worksheet for Week 3: Graphs of $f(x)$ and $f'(x)$

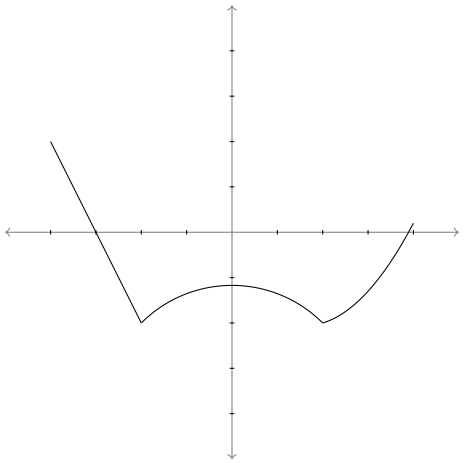
In this worksheet you'll practice getting information about a derivative from the graph of a function, and vice versa. At the end, you'll match some graphs of functions to graphs of their derivatives.

If $f(x)$ is a function, then remember that we define

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

If this limit exists, then $f'(x)$ is the slope of the tangent line to the graph of f at the point $(x, f(x))$.

Consider the graph of $f(x)$ below:



1. Use the graph to answer the following questions.
   (a) Are there any values x for which the derivative $f'(x)$ does *not* exist?

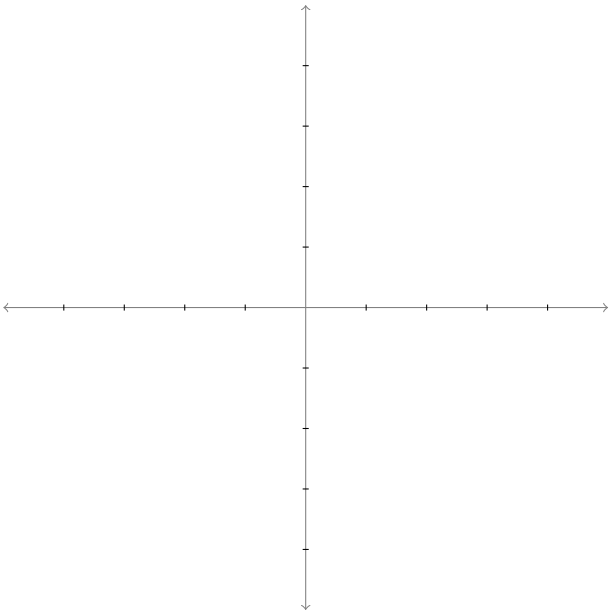
   (b) Are there any values x for which $f'(x) = 0$?

(c) This particular function f has an interval on which its derivative $f'(x)$ is constant. What is this interval? What does the derivative function look like there? Estimate the slope of $f(x)$ on that interval.

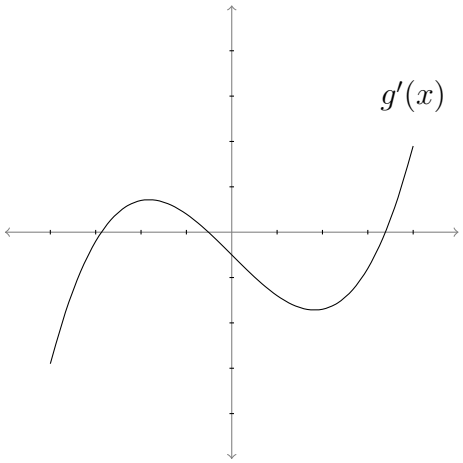
(d) On which interval or intervals is $f'(x)$ positive?

(e) On which interval or intervals is $f'(x)$ negative? Again, sketch a graph of the derivative on those intervals.

(f) Now use all your answers to the questions to sketch a graph of the derivative function $f'(x)$ on the coordinate plane below.



2. Below is a graph of a derivative $g'(x)$. Assume this is the entire graph of $g'(x)$. Use the graph to answer the following questions about the original function $g(x)$.



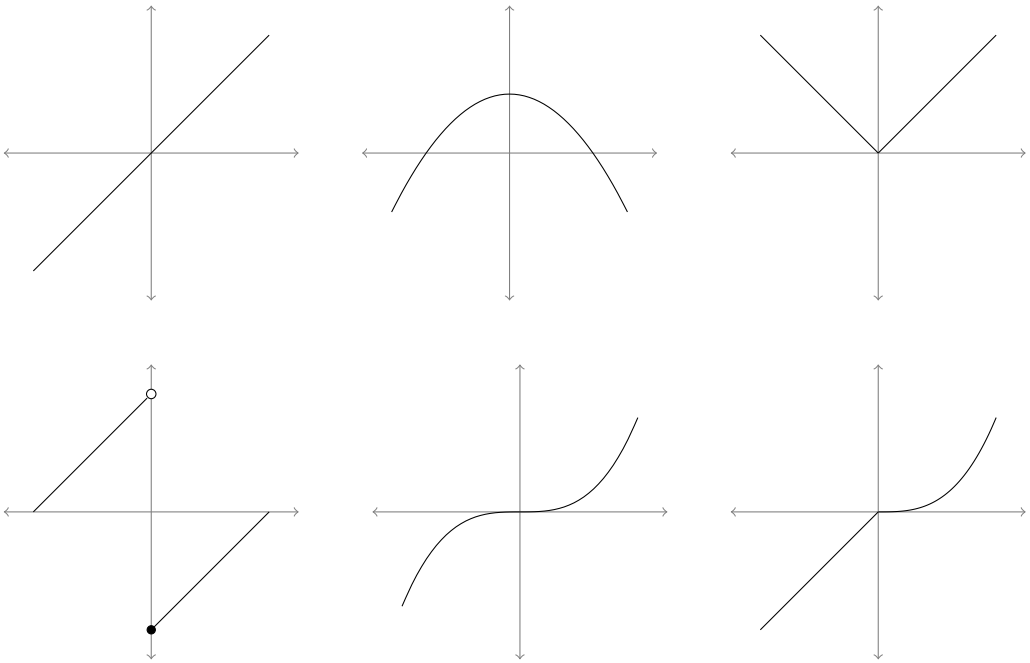
(a) On which interval or intervals is the original function $g(x)$ increasing?

(b) On which interval or intervals is the original function $g(x)$ decreasing?

(c) Now suppose $g(0) = 0$. Is the function $g(x)$ ever positive? That is, is there any x so that $g(x) \gneq 0$? How do you know?

3. Six graphs of functions are below, along with six graphs of derivatives. Match the graph of each function with the graph of its derivative.

Original Functions:



Their derivatives: