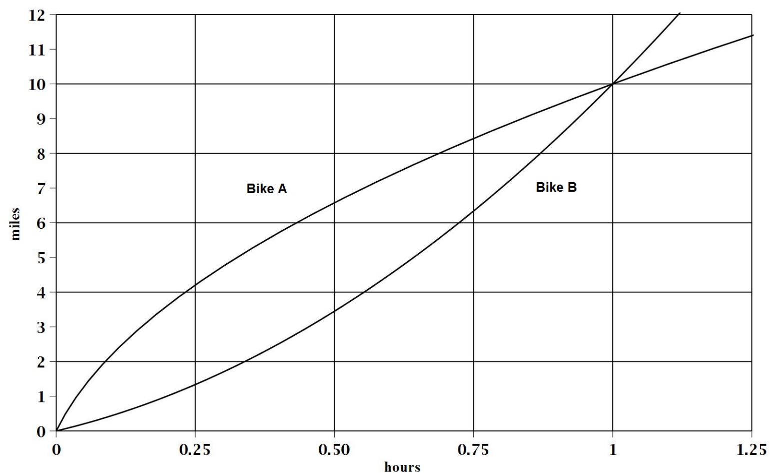
Worksheet for Week 2: Distance and Speed

Speed is the **rate of change** of distance. In this worksheet we look at this relationship using graphs. Since speed is the rate of change of distance, on the distance graph it should be related to a **slope**.

1. Consider the graph below, which shows how the positions of two bicycles change as time passes.



(a) Compute the average speed of Bike A from 15 to 30 minutes (so 0.25 to 0.5 hours). The answer will not be exact as you have only a graph and not the actual equations for the position functions.

> **Solution:** This is the slope of the secant line through points with $x = 0.25$ and $x = 0.5$, which is about 9.5 miles per hour.

(b) Now draw the line whose slope represents the speed you computed above on the graph.

> **Solution:** See graph at the end of the question.

(c) Compute the average speed of Bike B from 30 to 31 minutes. This may be more difficult to do than part (a). Why?

> **Solution:** This is the slope of the secant line through points with $x = 30/60$ and $x = 31/60$, which is about 10.1 miles per hour as given in part (e). Using points further away on the line for computing its slope (instead of the original points given) would have reduced your error.

(d) Now draw the line whose slope is the speed you computed in part (c). You can compute the slope of a line using any two points on the line. Recompute the average speed of Bike B from 30 to 31 minutes by computing the slope of that line as best as you can.

Solution: This is also the slope of the secant line through points with $x = 30/60$ and $x = 31/60$, which is about 10.1 miles per hour as given in part (e).

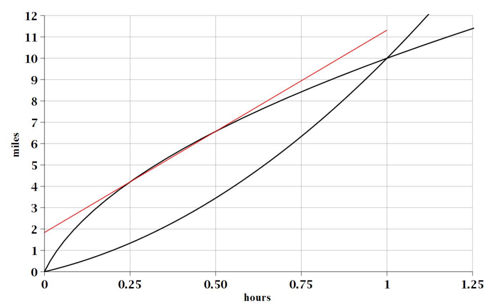
(e) The correct answer to parts (c) and (d) is approximately 10.1 miles per hour. Which of your answers above was closer? Why?

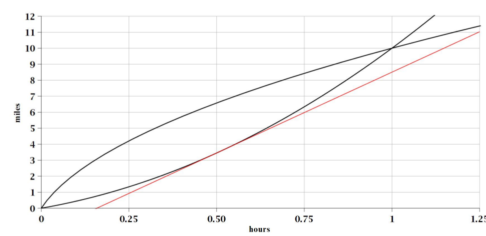
Solution: Using points further away on the line for computing its slope (instead of the original points given) would have reduced your error.

(f) If you want to compute the speed of Bike A at 30 minutes, what can you do? How is this related to a slope?

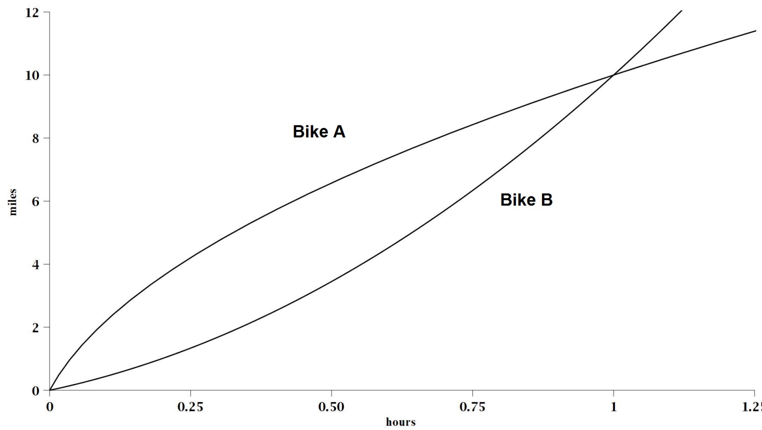
Solution: The speed of Bike A at 30 minutes would be the slope of the **tangent line** at 30 minutes, so at $x = 0.5$.

Solution: Below are the two secants you needed for the first four parts. How do your lines compare?





2. Here is the graph again. This time you will not be doing numerical computations so the grid lines have been removed for a cleaner look.



(a) Which bike is moving faster at 15 minutes? How do you know?

> **Solution:** Bike A; its position is changing more quickly. The slope of the tangent line at $x = 0.25$ is steeper on the distance function for Bike A.

(b) Which bike is moving faster at at $t = 1$?

> **Solution:** Bike B is moving faster because the slope of the tangent line at $x = 1$ is steeper on the distance function for Bike B.

(c) At the end of one hour which bike is ahead? How can you tell?

> **Solution:** They are at the same place since their distance functions have the same value (10 miles).

3. Notice that a steeper curve on the graph corresponds to a higher velocity. A steep curve means that the position is changing quickly, which means the bike is moving fast. Refer to the graph on the previous page to answer the following questions.

   (a) According to the graph, during the second half-hour of the bike ride, when is Bike A moving the fastest?

   Solution: Bike A slows down throughout its journey. The slopes of the tangents are decreasing. So it is fastest at $t = 0.5$ hours.

   (b) At about what time does Bike B start catching up with Bike A? That is, when does the distance between the bikes start to shrink?

   Solution: About 0.4 hours. That is when the distance between them is maximum.

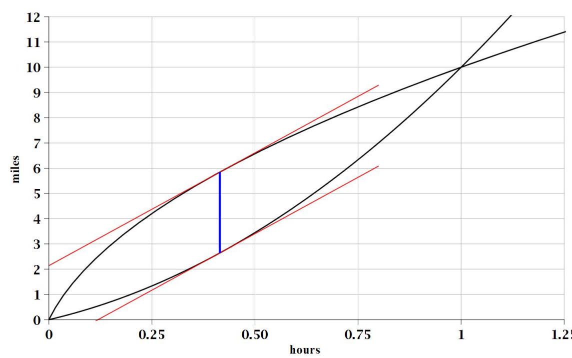
   (c) Do you think there is an a time when the bikes are moving at exactly the same velocity? Either estimate that time by looking at the graph, or explain why there can't be such a time.

   Solution: Rephrase the question: is there a time t so that the tangent line to the curve for A is parallel to the tangent line to the curve for B? So, yes, at about 0.4 hours.

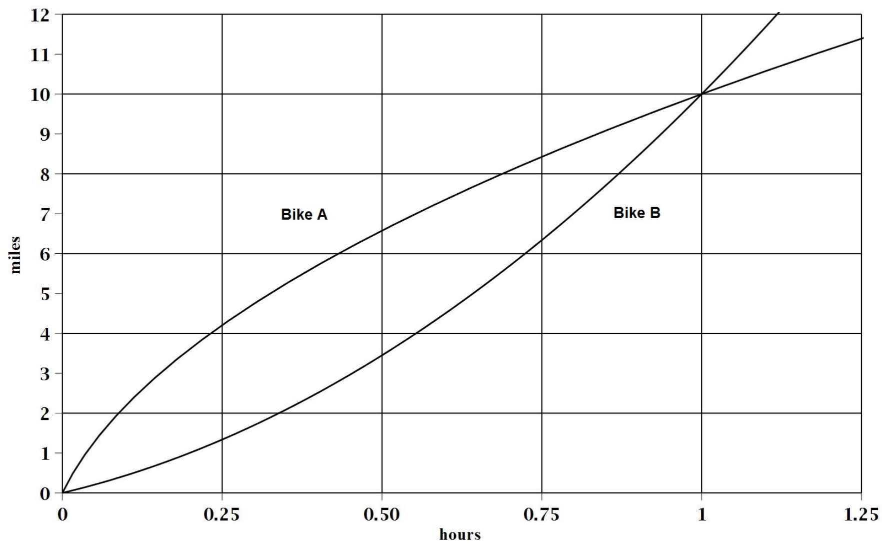
   (d) Are the questions and answers to parts (3b) and (3c) related? Why or why not?

   Solution: They are exactly the same. The distance between them will start to shrink right at the moment when it is a maximum. That will happen exactly when Bike B is about the go faster than Bike A, starting to catch up with him. Take a moment and imagine yourself on a bike trying to catch up to the person in front of you.

   Below is the graphs with the two tangents with the same slope when the distance between them is maximum:

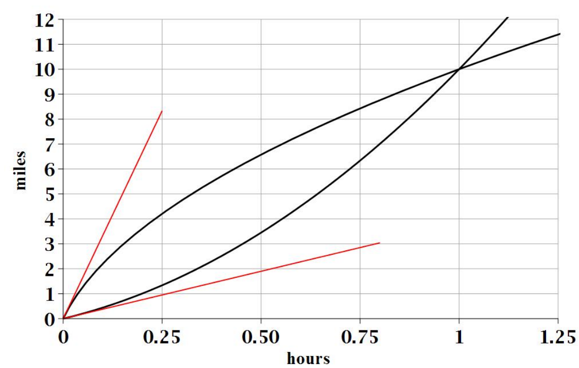


4. At the end of this question you will sketch graphs of their speeds. First, we will start collecting some information. Here is the graph from the first page - again with grids for easier computation.



(a) Calculate their speeds when they start out as best as you can. By now you know these have to be slopes of tangent lines at the origin. Make sure you draw your tangents carefully.

Solution: Bike A starts at about 33 miles per hour and Bike B starts at about 3.8 miles per hour. They are the slopes of the tangent lines below.



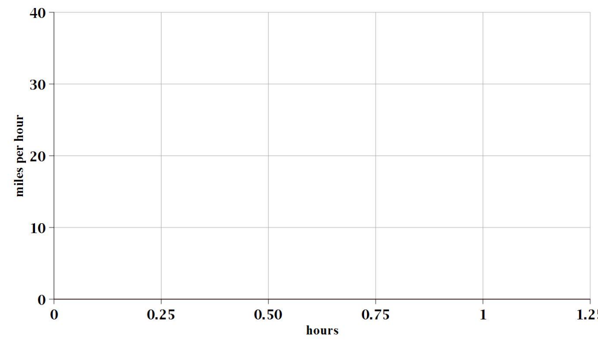
(b) Calculate their common speed at the time you found in Question 3(c).

Solution: About 8.9 miles per hour, the slopes of the lines in 3(d) above.

(c) Approximate their speeds at the end of one hour.

Solution: Bike B going at about 16.2 miles per hour and Bike A going at about 5.9 miles per hour.

(d) Are their speeds increasing or decreasing through the journey. Use the information you collected to sketch their speed graphs on the right. This will be very approximate. You can compare your answers with the actual speed graphs in the solutions later.



Solution: The Speed of Bike A decreases, while the speed of Bike B increases. Your graphs should look (more or less) like the ones on the right: One decreasing, the other increasing, meeting at $x = 0.4$, starting at the values in part (a), ending at the values in part (c). You cannot get their exact shapes - and the fact that one is a line, the other is not- from what you are given. These graphs were generated using the equations not provided in the question.