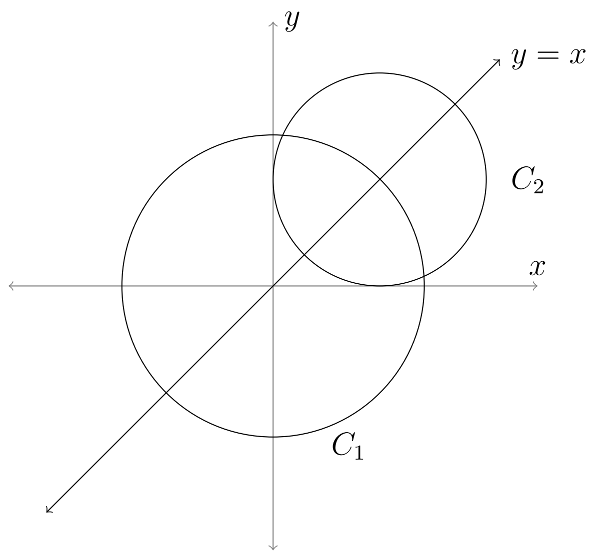
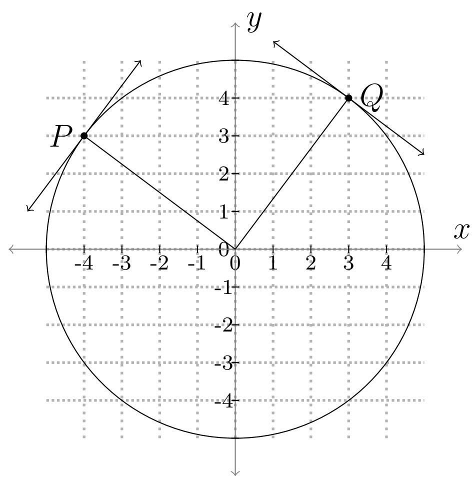
Worksheet for Week 1: Circles and lines

This worksheet is a review of circles and lines, and will give you some practice with algebra and with graphing. Also, this worksheet introduces the idea of "tangent lines" to circles. Later on in Math 124, you'll learn how to find tangent lines to many other types of curves.

1. Two circles, called C_1 and C_2, are graphed below. The center of C_1 is at the origin, and the center of C_2 is the point in the first quadrant where the line $y = x$ intersects C_1. Suppose C_1 has radius 2. C_2 touches the x and y axes each in one point. What are the equations of the two circles?

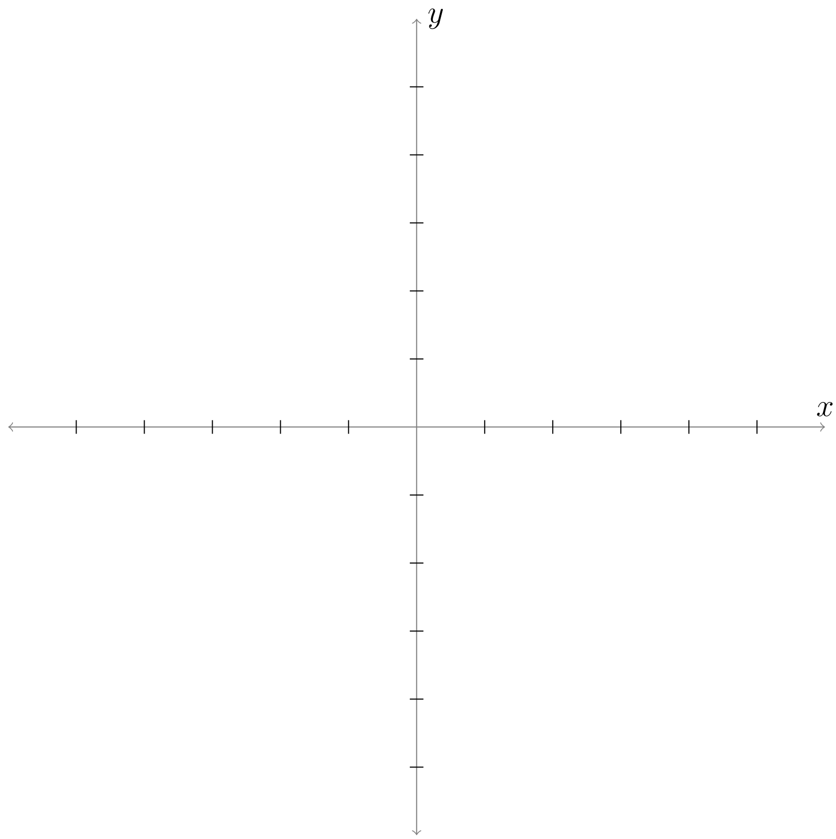


2. Let C be the circle of radius 5 centered at the origin. The **tangent line** to C at a point Q is the line through Q that's perpendicular to the radial line connecting Q to the center. (See picture.) Use this information to find the equations of the tangent lines at P and Q below.



Note: Later in Math 124, you'll learn how to find tangent lines to curves that are not circles!

3. Sketch the circle of radius 2 centered at $(3, -3)$ and the line L with equation $y = 2x + 2$. Find the coordinates of all the points on the circle where the tangent line is perpendicular to L.

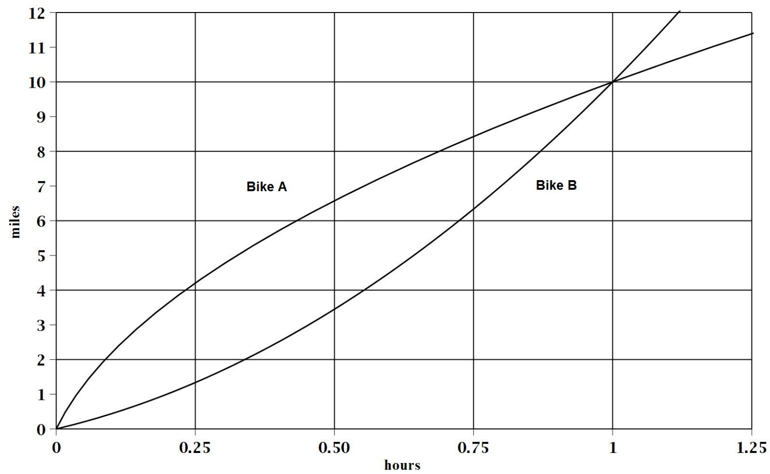


4. Draw the circle with equation $x^2+y^2 = 25$ and the points $P = (-3, -4)$ and $Q = (-8, 0)$. Explain why P is on the circle. Is the line through P and Q tangent to the circle? How do you know?

Worksheet for Week 2: Distance and Speed

Speed is the **rate of change** of distance. In this worksheet we look at this relationship using graphs. Since speed is the rate of change of distance, on the distance graph it should be related to a **slope**.

1. Consider the graph below, which shows how the positions of two bicycles change as time passes.



   (a) Compute the average speed of Bike A from 15 to 30 minutes (so 0.25 to 0.5 hours). The answer will not be exact as you have only a graph and not the actual equations for the position functions.

   (b) Now draw the line whose slope represents the speed you computed above on the graph.

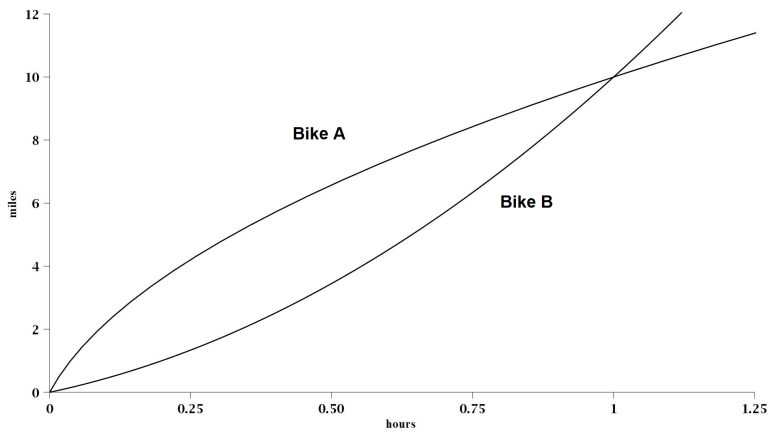
   (c) Compute the average speed of Bike B from 30 to 31 minutes. This may be more difficult to do than part (a). Why?

(d) Now draw the line whose slope is the speed you computed in part (c). You can compute the slope of a line using any two points on the line. Recompute the average speed of Bike B from 30 to 31 minutes by computing the slope of that line as best as you can.

(e) The correct answer to parts (c) and (d) is approximately 10.1 miles per hour. Which of your answers above was closer? Why?

(f) If you want to compute the speed of Bike A at 30 minutes, what can you do? How is this related to a slope?

2. Here is the graph again. This time you will not be doing numerical computations so the grid lines have been removed for a cleaner look.



(a) Which bike is moving faster at 15 minutes? How do you know?

(b) Which bike is moving faster at at $t = 1$?

(c) At the end of one hour which bike is ahead? How can you tell?

3. Notice that a steeper curve on the graph corresponds to a higher velocity. A steep curve means that the position is changing quickly, which means the bike is moving fast. Refer to the graph on the previous page to answer the following questions.

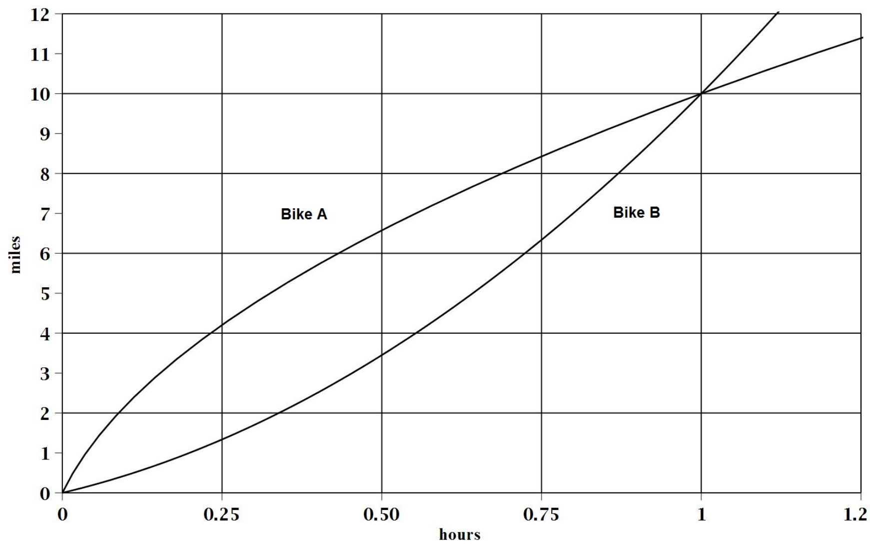
   (a) According to the graph, during the second half-hour of the bike ride, when is Bike A moving the fastest?

   (b) At about what time does Bike B start catching up with Bike A? That is, when does the distance between the bikes start to shrink?

   (c) Do you think there is an a time when the bikes are moving at exactly the same velocity? Either estimate that time by looking at the graph, or explain why there can't be such a time.

   (d) Are the questions and answers to parts (3b) and (3c) related? Why or why not?

4. At the end of this question you will sketch graphs of their speeds. First, we will start collecting some information. Here is the graph from the first page - again with grids for easier computation.

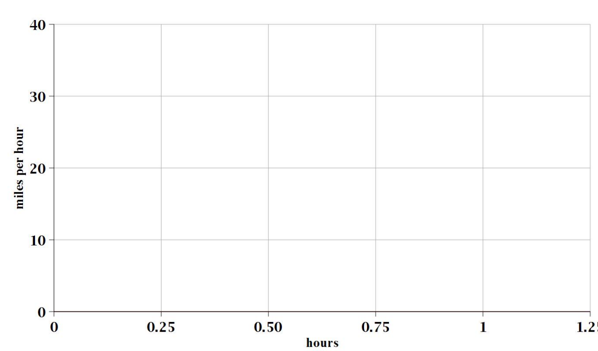


   (a) Calculate their speeds when they start out as best as you can. By now you know these have to be slopes of tangent lines at the origin. Make sure you draw your tangents carefully.

   (b) Calculate their common speed at the time you found in Question 3(c).

   (c) Approximate their speeds at the end of one hour.

   (d) Are their speeds increasing or decreasing through the journey. Use the information you collected to sketch their speed graphs on the right. This will be very approximate. You can compare your answers with the actual speed graphs in the solutions later.



Worksheet for Week 3: Graphs of $f(x)$ and $f'(x)$

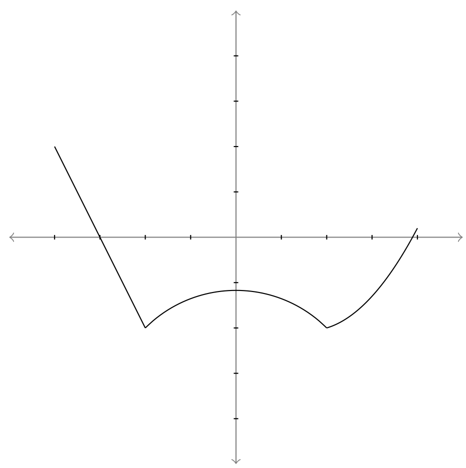
In this worksheet you'll practice getting information about a derivative from the graph of a function, and vice versa. At the end, you'll match some graphs of functions to graphs of their derivatives.

If $f(x)$ is a function, then remember that we define

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

If this limit exists, then $f'(x)$ is the slope of the tangent line to the graph of f at the point $(x, f(x))$.

Consider the graph of $f(x)$ below:



1. Use the graph to answer the following questions.
   (a) Are there any values x for which the derivative $f'(x)$ does *not* exist?

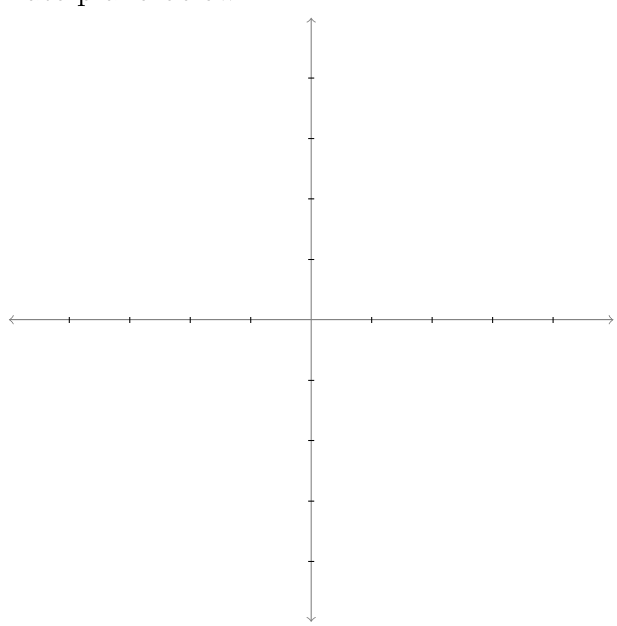
   (b) Are there any values x for which $f'(x) = 0$?

(c) This particular function f has an interval on which its derivative $f'(x)$ is constant. What is this interval? What does the derivative function look like there? Estimate the slope of $f(x)$ on that interval.

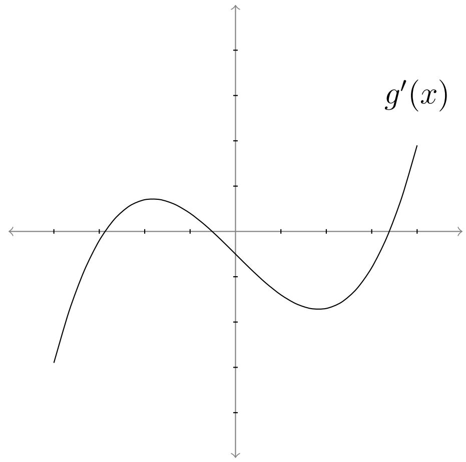
(d) On which interval or intervals is $f'(x)$ positive?

(e) On which interval or intervals is $f'(x)$ negative? Again, sketch a graph of the derivative on those intervals.

(f) Now use all your answers to the questions to sketch a graph of the derivative function $f'(x)$ on the coordinate plane below.



2. Below is a graph of a derivative $g'(x)$. Assume this is the entire graph of $g'(x)$. Use the graph to answer the following questions about the original function $g(x)$.



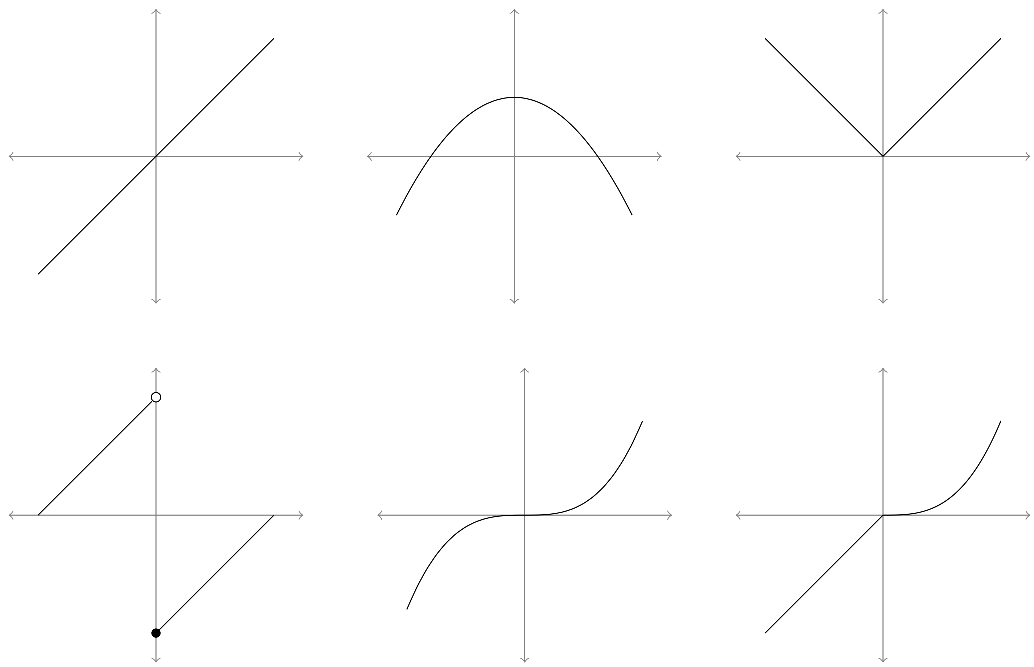
(a) On which interval or intervals is the original function $g(x)$ increasing?

(b) On which interval or intervals is the original function $g(x)$ decreasing?

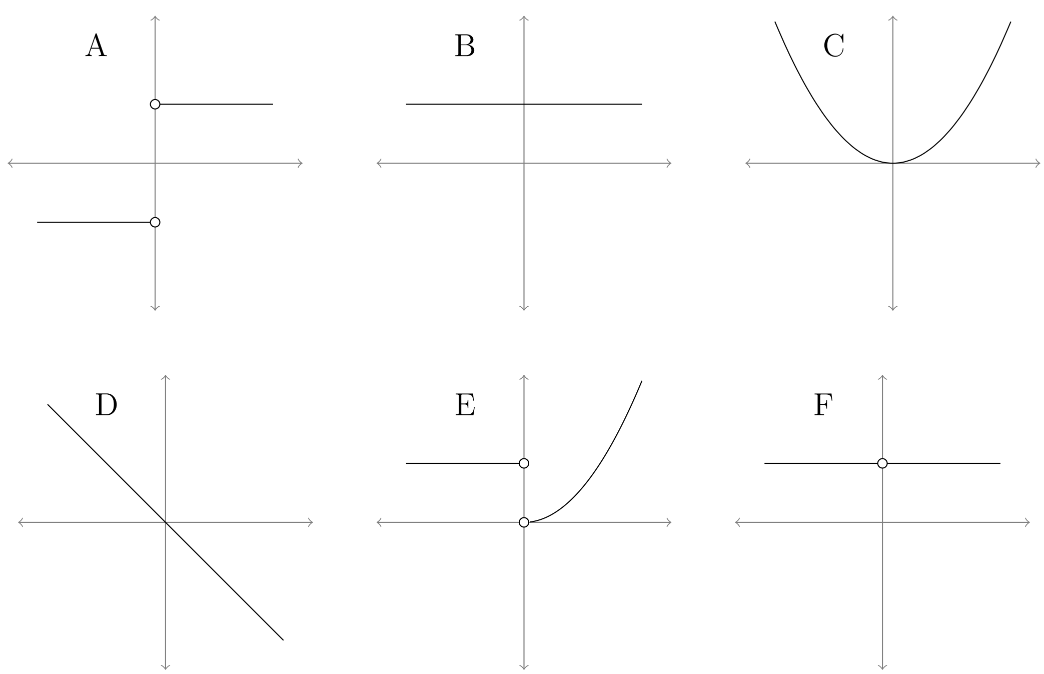
(c) Now suppose $g(0) = 0$. Is the function $g(x)$ ever positive? That is, is there any x so that $g(x) \gneq 0$? How do you know?

3. Six graphs of functions are below, along with six graphs of derivatives. Match the graph of each function with the graph of its derivative.

Original Functions:



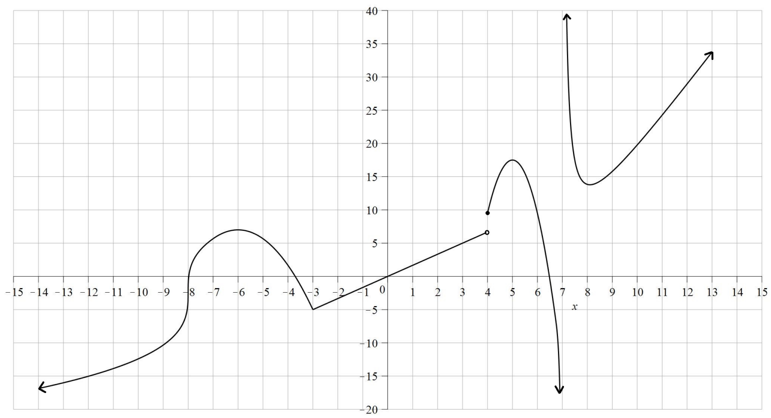
Their derivatives:



Worksheet for Week 4: Limits and Derivatives

This worksheet reviews limits and the definition of the derivative with graphs and computations.

1. Answer the following questions using the graph $y = f(x)$ below. The function $f(x)$ has domain all numbers except 7 as seen from the graph.



(a) $\lim_{x\to 4} f(x) =$

(b) $\lim_{x\to 7^+} f(x) =$

(c) $f'(0) =$

(d) $\lim_{x\to -3} f(x) =$

(e) $\lim_{x\to 0} \frac{f(x)}{x} =$

(f) $\lim_{h\to 0} \frac{f(3+h)-5}{h} =$

(g) $f'(5) =$

(h) $\lim_{h\to 0^+} \frac{f(-8+h)-f(-8)}{h} =$

(i) $\lim_{h\to 0} \frac{f(-8+h)}{h} =$

(j) $\lim_{h\to 0} \frac{f(-6+h)-f(-6)}{h} =$

(k) $\lim_{h\to 0^+} \frac{f(-3+h)+5}{h} =$

(l) List all the intervals where the derivative $f'(x)$ is negative.

(m) List all the intervals where the derivative $f'(x)$ is decreasing.

(n) A critical value for $f(x)$ is any x in the domain of $f(x)$ where $f'(x) = 0$ or $f'(x)$ is undefined. List all critical values of $f(x)$.

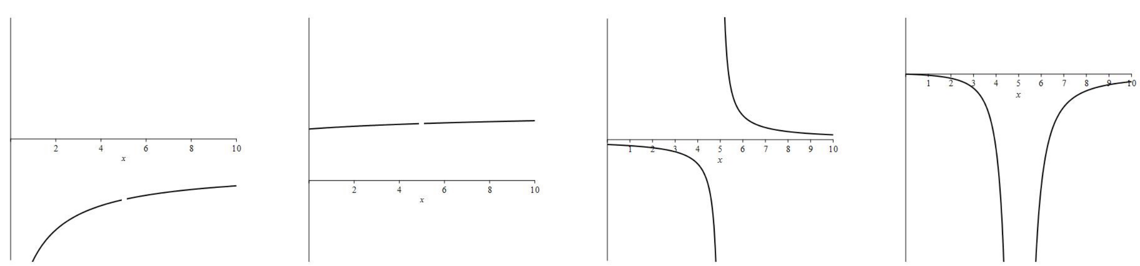
2. Evaluate the following limits and then match the functions with their graphs shown below using your limit results. Some will require you to compute left and right hand limits.

   (a) $\lim_{x \to 5} \dfrac{1}{x-5} =$

   (b) $\lim_{x \to 5} \dfrac{-x}{(x-5)^2} =$

   (c) $\lim_{x \to 5} \dfrac{-x^2 - 2x + 35}{x^2 - 4x - 5} =$

   (d) $\lim_{x \to 5} \dfrac{x - \sqrt{3x+10}}{x-5} =$

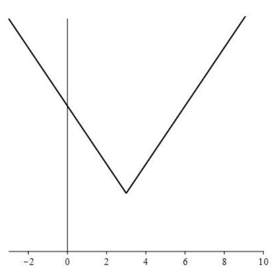


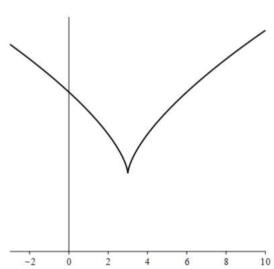
3. Use the definition of the derivative $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ to compute $f'(3)$ for the following functions. Then match the functions with their graphs shown below using your limit results.

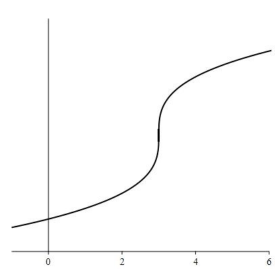
   (a) $f(x) = (x-3)^{\frac{1}{3}} + 2$

   (b) $f(x) = (x-3)^{\frac{2}{3}} + 2$

   (c) $f(x) = |x-3| + 2$







4. Find, if any, the horizontal asymptotes of the following functions and use that information to match them with their graphs on the next page. Each question should have two limit computations with $x \to \infty$ and $x \to -\infty$.

   (a) $f(x) = \dfrac{(x+1)^4}{x^4 + 3x^2 + 7x + 10}$

   (b) $f(x) = \dfrac{x+3}{x^2 + 8x + 26}$

   (c) $f(x) = \dfrac{x^3 + 4x + 9}{x^2 + 4}$

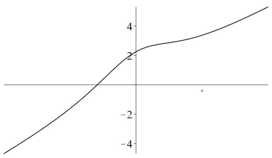
(d) $f(x) = -7x^4 + x^3 - 12x + 20$

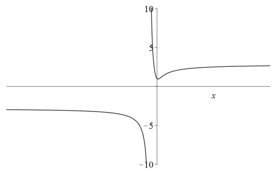
(e) $f(x) = \dfrac{\sqrt{8x^2 + 4}}{x + 2}$

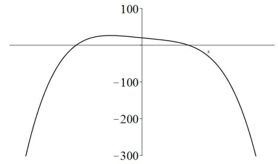
(f) $f(x) = 3e^x$

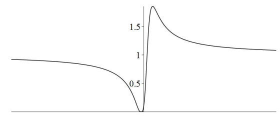
(g) $f(x) = 7 - e^{-x}$

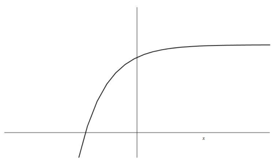
When you match the functions with these graphs, add (if any) horizontal asymptotes to the pictures.

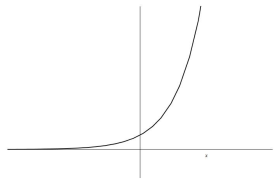


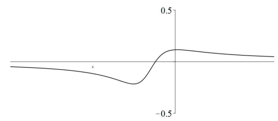






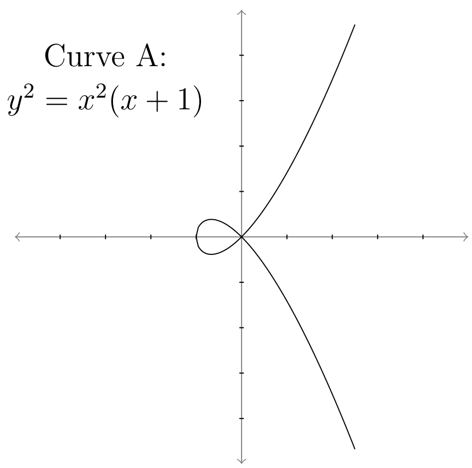


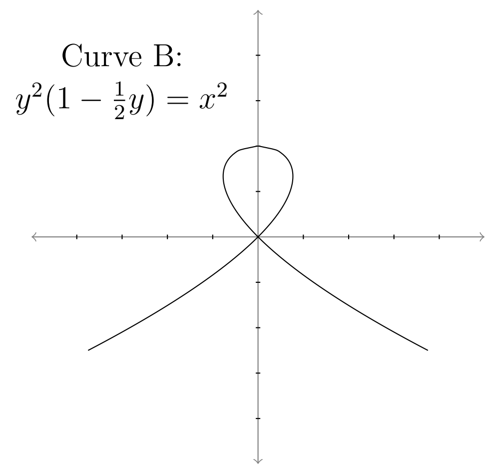




Worksheet for Week 6: Implicit Differentiation and Parametric Curves

In this worksheet, you'll use parametrization to deal with curves that have more than one tangent line at a point. Then you'll use implicit differentiation to relate two derivative functions, and solve for one using given information about the other.





1. (a) Use impicit differentiation to find all the points in Curve A with a horizontal tangent line. (Looking at the graph, how many such points should there be?)

   (b) What about Curve B?

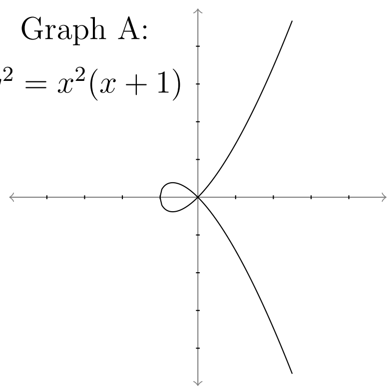
   (c) Try to find $\frac{dy}{dx}$ at the point $(0,0)$ on both graphs. What goes wrong?

2. In this problem you'll look at the curves from Page 1 in a different way.

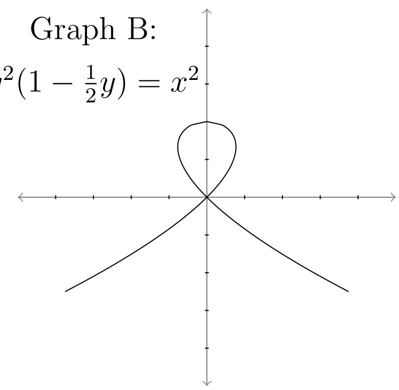
   Suppose a cat is chasing a ball around on the floor, and its position is described by the parametric equations

   $$(x(t), y(t)) = (t^2 - 1, t - t^3).$$

   Graph A: $y^2 = x^2(x+1)$

   Graph B: $y^2(1 - \frac{1}{2}y) = x^2$

   (a) The cat is following one of the paths from the previous page (reprinted above). Which path does the cat follow? Circle this curve. How do you know it's the right one?

   (b) Draw an arrow on the circled graph above, to indicate in which direction the cat is running.

(c) At which time(s) t does the cat run through the point $(0, 0)$?

(d) Remember that it wasn't possible to find $\frac{dy}{dx}$ at $(0, 0)$ using the method on Page 1. But now that the graph has been parametrized, you can do it. What are the tangent line(s) to the parametrized curve $(x(t), y(t))$ at $(0, 0)$?

3. This next question is a new type of problem that you can solve now that you know about implicit differentiation. Suppose a snowball is rolling down a hill, and its radius r is growing at a rate of 1 inch per minute. The volume V of the snowball grows more quickly as the snowball gets bigger. In this question, you'll find the rate of change of the volume, $\frac{dV}{dt}$, at the instant when the radius r is 6 inches.

   (a) First, apply geometry to the situation. Can you think of an equation that relates the variables r and V to each other?

   (b) Now the variables V and r change as time changes, so we can think of them as functions of t. Differentiate the equation you came up with in part (a) with respect to t.

   (c) What is the rate of change of the radius? Use this to simplify your equation from part (b).

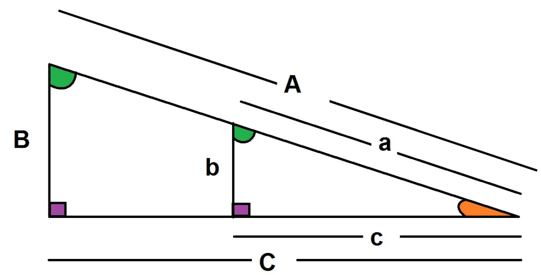
   (d) What is the rate of change of V when the radius of the snowball is 6 inches?

Worksheet for Week 7: Geometry Review

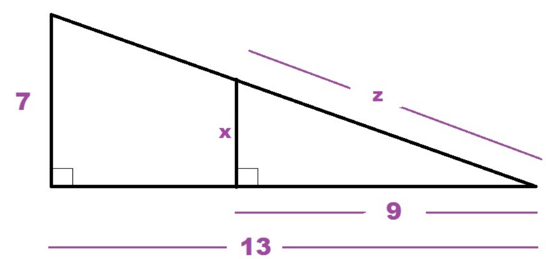
Many of the story problems will require writing down functions using **geometry**. You need **area** and **volume** equations, the **Pythagorean Theorem**, definitions of the **trigonometric functions** on the right triangle and **similar triangles**. This worksheet covers geometry of triangles.

1. **Using Similar Triangles**
   Similar triangles have a common ratio for their corresponding sides, opposite the angles with the same measures. In particular, when one right triangle sits inside another, the two triangles are similar so you get the ratios:

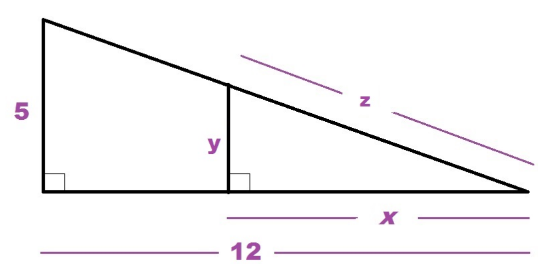
$$\frac{A}{a} = \frac{B}{b} = \frac{C}{c}$$



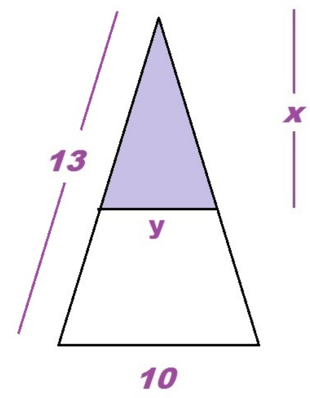
(a) Use the fact that two right triangles are similar to compute x and z.



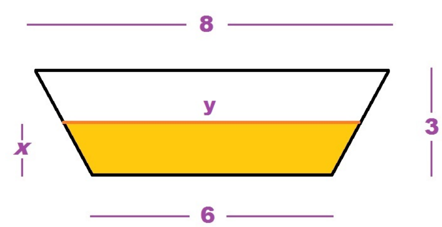
(b) Use the fact that two right triangles are similar to compute y and z in terms of x.



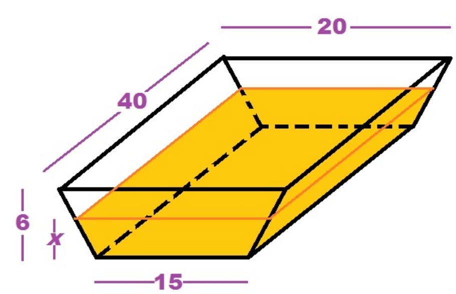
(c) Find y in terms of x. You need right triangles to get started on this one.



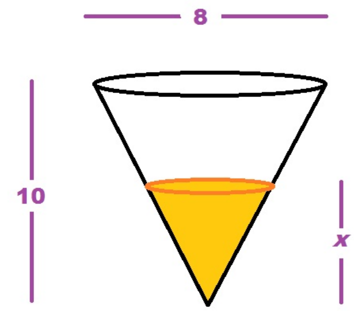
(d) Find the area of the shaded region in terms of x. You need right triangles to get started on this one, too.



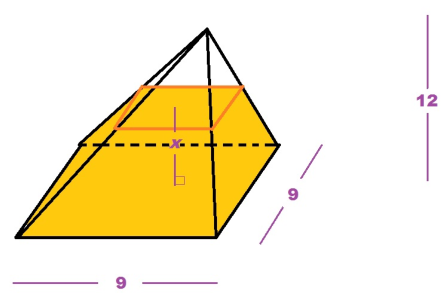
(e) The tank shown on the right is filled with water to a depth x. Find the volume of water in the tank in terms of x and the total volume of the tank. Remember to compute the volume you multiply base area by height. Which one is the base?



(f) The tank shown on the right is filled with water to a depth x. Find the volume of water in the tank in terms of x and the total volume of the tank. What is the formula for the volume of a cone?

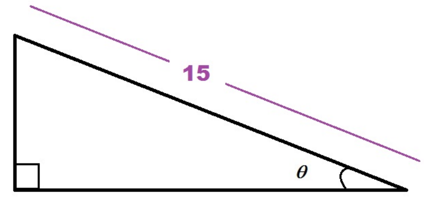


(g) The tank shown on the right is filled with water to a depth x. Find the total volume of the tank and the volume of water in the tank in terms of x. What is the formula for the volume of a pyramid?



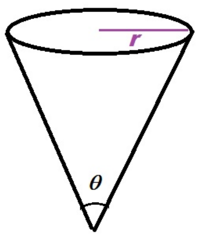
2. **Circles and Angles -** For this question you will need the trigonometric functions. Remember that we always use RADIANS for angles in calculus.

   (a) Find the area and the perimeter of the triangle.

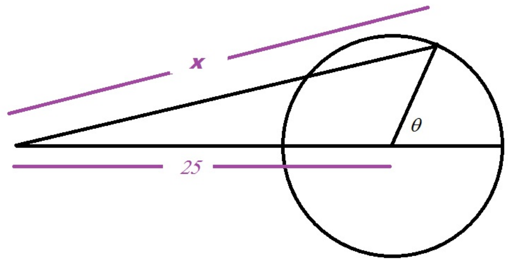


   (b) Find the area and the perimeter of the trapezoid.

   (c) Find the volume of the cone in terms of r and θ.



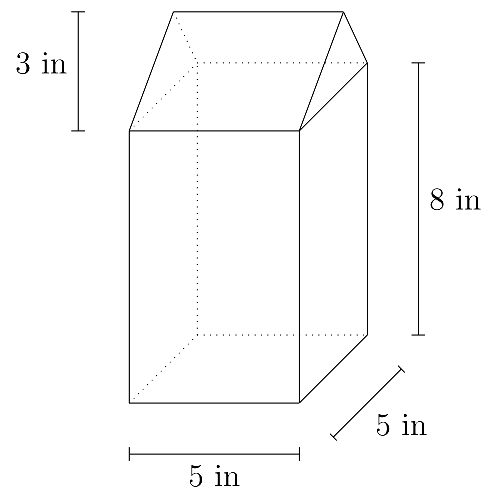
   (d) Find x given that the radius of the circle is 5 units long.



Worksheet for Week 8: Related Rates

This worksheet guides you through some more challenging problems about related rates.

A milk carton is shaped like a tall box with a triangular prism on top. The sides of the top section are isosceles triangles. This particular milk carton has a 5 inch × 5 inch square base, and is 11 inches tall. (See the picture.)



Suppose you're filling the carton with liquid, at a rate of 10 inches^3 per minute. In this problem, you'll figure out the rate of change of the height of the liquid in the carton, at the instant when the carton holds 220 inches^3 of liquid.

1. (a) Let y be the height of the liquid in the carton. Then y is a function of time, because y is changing as more liquid pours in. Suppose $y \leq 8$, so that the liquid is all in the rectangular part of the carton. Find a formula for the total volume of liquid in the carton.

   (b) Now suppose $8 \leq y \leq 11$, so that some liquid is in the triangular part of the carton. Find a formula for the total volume of liquid in the carton, in terms of y. You might want to break up the volume into two pieces: the volume below the 8-inch line and the volume above the 8-inch line.

(c) Next, suppose that 220 in^3 of liquid is in the carton. How high is the liquid level? That is, what is y?

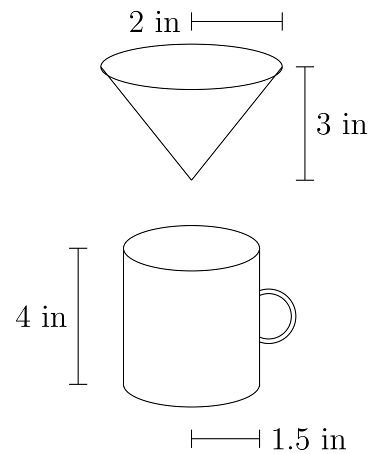
(d) What is $\frac{dy}{dt}$ when 220 in^3 of liquid is in the carton?

(e) When y reaches 8 inches, does $\frac{dy}{dt}$ increase, decrease, or stay the same? You can answer this question by using the formulas you found on Page 1, or by looking at the picture. How do you know your answer is correct?

2. Now suppose a funnel is positioned over a coffee mug.

   The funnel is a cone 3 inches high with a base radius of 2 inches, and the coffee mug is 4 inches tall with a 1.5-inch radius. See the picture at right.

   Suppose the funnel is initially **full** of coffee. Then it starts to drip down into the mug at a constant rate of 0.1 inches^3 per second.



   (a) Let y be the height of the coffee in the funnel at any given time, so that just before the dripping starts we have $y = 3$. Since the coffee is draining out of the funnel, $\frac{dy}{dt}$ will be negative. What (approximately) will be the value of y when $\frac{dy}{dt}$ is at its smallest (closest to zero)?

   (b) What (approximately) will y be when $\frac{dy}{dt}$ is biggest (farthest from zero)?

   (c) How much coffee is in the funnel at the very beginning?

   (d) How much coffee is in the funnel **and mug** at some time t?

(e) Find a formula, depending on the height y of coffee in the funnel, for the volume of coffee in the funnel.

(f) Now suppose the coffee **mug** is one-third full of coffee. How fast is the height of coffee in the funnel changing? In other words, what is $\frac{dy}{dt}$ at that instant?

Worksheet for Week 9: Indeterminate Forms and L'Hospital's Rule

We worked with limits at the beginning of the quarter, sometimes using algebraic tricks to compute them. Now, we return to limits which have *indeterminate forms* and use L'Hospital's Rule to evaluate them. Although we have seven indeterminate forms

$$\frac{"\infty"}{\infty}, \quad \frac{"0"}{0}, \quad "\infty - \infty", "\infty \cdot 0", \quad "1^{\infty}", \quad "0^{0}", \quad "\infty^{0}"$$

(all in quotes because they are not actual mathematical expressions to be evaluated), L'Hospital's Rule only works with indeterminate quotients.

L'Hospital's Rule

Suppose f and g are differentiable functions and $g'(x)$ is not identically zero near a, except possibly at $x = a$. Suppose that

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$$

or

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = \pm\infty.$$

Then,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

if the limit on the right exists (or is ∞ or $-\infty$).

The rule is actually more general and it works with limits $x \to \infty$, $x \to -\infty$, $x \to a^-$ and $x \to a^+$, as well.

It is important that you have an $\frac{"\infty"}{\infty}$ or $\frac{"0"}{0}$ **indeterminate form before you use L'Hospital's Rule.**

1. **Indeterminate Quotients and L'Hospital's Rule**

   For each of the limits below, determine if it is the indeterminate quotient $\frac{"\infty"}{\infty}$, the indeterminate quotient $\frac{"0"}{0}$, or if it is not indeterminate. If you have an indeterminate quotient evaluate the limit using L'Hospital's Rule. If the limit is not indeterminate, you should be able to evaluate it without much effort.

   (a) $\lim_{x \to -\infty} \frac{1+\sqrt{3-x}}{x}$

   (b) $\lim_{x \to 0} \frac{\sin x - x}{x^3} =$

   (c) $\lim_{x \to 1} \frac{x^3 - x}{x^3 - 2x^2 - 3x} =$

(d) $\lim_{x \to 2} \frac{x^2 + x - 6}{x^3 - 4x^2 + 4x}$

(e) $\lim_{x \to \infty} \frac{\ln x}{x^a}$ (*Be careful. Your answer will depend on a.*)

(f) $\lim_{x \to 0} \frac{x - \sin^{-1} x}{x^3}$

2. **L'Hospital may not always be the best way to go**
The following limits are all indeterminate quotients. Although L'Hospital works on most (but not all!) of them, it may not be the best approach. Some of the limits, try to remember older tricks from the second week of the course for evaluating limits. For others, you may want to split up and work in pieces.

   (a) $\lim_{x\to\infty} \frac{\sqrt{1+x^2}}{3+2x}$

   (b) $\lim_{x\to\infty} \frac{x+\cos x}{x+\sin x}$

(c) $\lim_{x \to 0} \frac{\sin^4(x)\sin^6(3x)}{5x^{10}}$

(d) $\lim_{x \to \infty} \frac{5x^2 3^{1+x} + 3x^4 \ln(x)}{2x^2 3^{3+x} + 9x^4 \ln(x)}$

3. **Indeterminate difference "$\infty - \infty$" and indeterminate product "$0 \cdot \infty$"**
Determine if the following limits are one of the indeterminate forms "$\infty - \infty$" or "$0 \cdot \infty$". If yes, state which one and use algebra to turn it into an indeterminate quotient to evaluate using L'Hospital Rule or some other idea. If the limit is not indeterminate, you should be able to evaluate it without much effort.

(a) $\lim_{u \to \infty} u \sin(1/u)$

(b) $\lim_{x \to \infty} \left(\sqrt{16x^2 + 4x} - 4x \right)$

(c) $\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

(d) $\lim_{x \to -\infty} \left(\sqrt{25x^2 + x} - 5x \right)$

4. **Indeterminate Powers "1^∞", "0^0", and "∞^0"**
Determine if the following limits are one of the indeterminate powers "1^∞", "0^0", or "∞^0". If yes, state which indeterminate power and evaluate by first using the natural logarithm function to bring the power down. (Which new indeterminate form do you have after taking the logarithm of both sides?)

(a) $\lim_{x\to 0^+} x^{\frac{1}{x}}$

(b) $\lim_{x\to 0^+} (\sin x)^x$

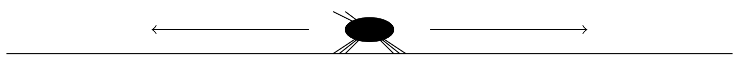
(c) $\lim_{x\to\infty} \left(\frac{x+b}{x-1}\right)^{x-1}$

Worksheet for Week 10: Sketching Curves

You might have wondered, *why bother learning how to sketch curves using calculus if I can just plug the equation into a computer and see the graph?* But it could happen that you don't actually have a neat formula for the function you're trying to graph. In this worksheet, you'll reconstruct a graph of a function given some data about it. There is no neat formula for the function, but calculus will help you figure out what it looks like anyway!

At the end of the worksheet, there is a problem about maximizing a function on a closed interval, for extra practice.

1. A scientist is watching a bug walk back and forth along a line. Suppose the line has a coordinate x, and let $p(t)$ be the continuous function giving the bug's position on the line at time t (in seconds).



The scientist observes the bug's motions and records what she sees:

- At $t = 0$, the bug was located at $x = -0.25$.
- At $t = 5$, the bug passed through the point $x = 0$ for the first and only time.
- As $t \to \infty$, the bug approaches $x = 0$. In other words, $\lim_{t\to\infty} p(t) = 0$.
- The derivative funtion $p'(t)$ — the velocity of the bug — is continuous. For a few seconds at the beginning $p'(t)$ was negative, but then it crossed 0 to become positive at $t = 3.3$. It crossed 0 to become negative again at $t = 6.7$, and remained negative thereafter.
- Also, $p'(t)$ had its maximum value at $t = 5$, and its most negative value at $t = 2.2$ and $t = 7.8$.
- The bug always stayed within 5 units of $x = 0$.

In this problem, you'll figure out how to sketch a graph of the bug's movements on the interval $[0, \infty)$, even though you don't know the formula for $p(t)$! For now, use the information above to answer the following questions. (*After* you answer them all, you'll get to make the sketch.)

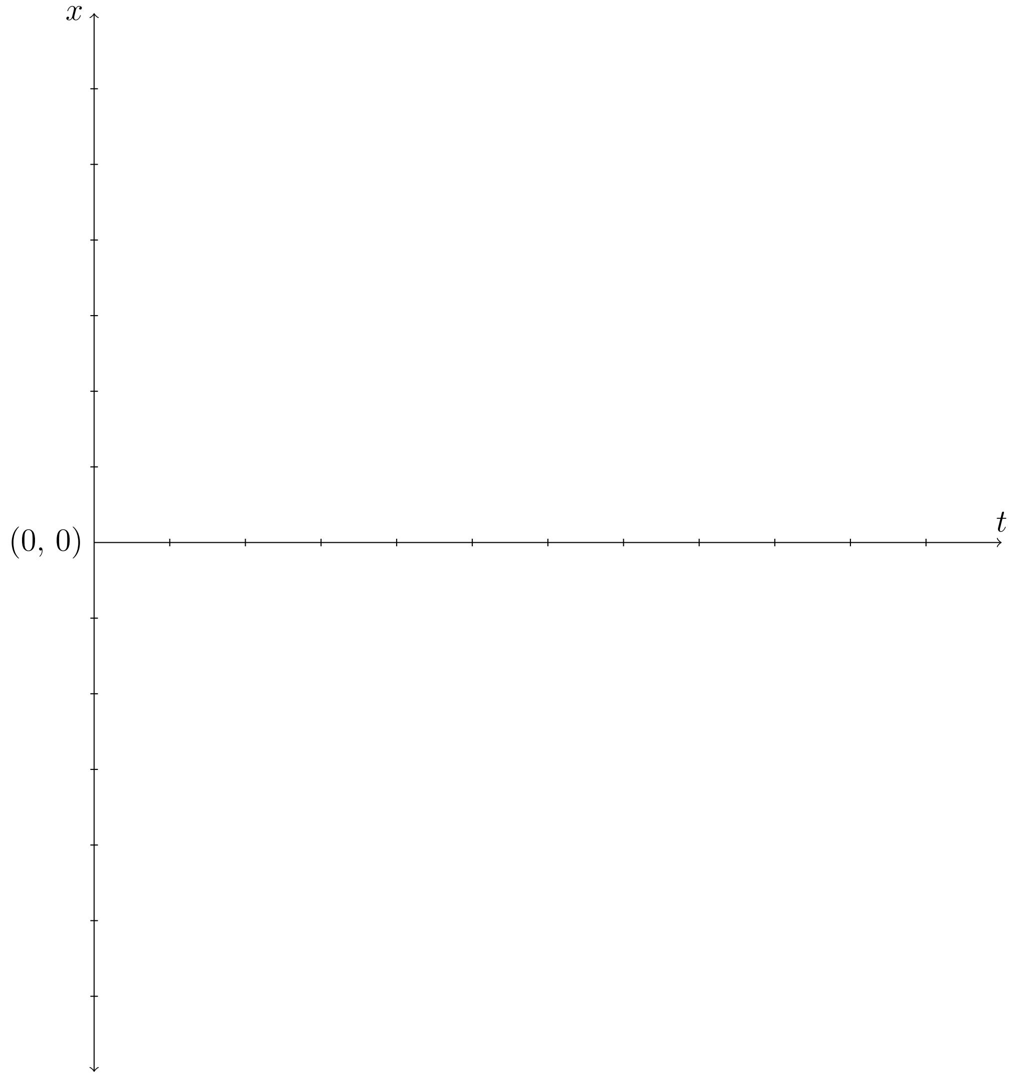
(a) Where will the graph of $p(t)$ intersect the t and x axes?

(b) Does the graph of $p(t)$ have any asymptotes? If so, where are they?

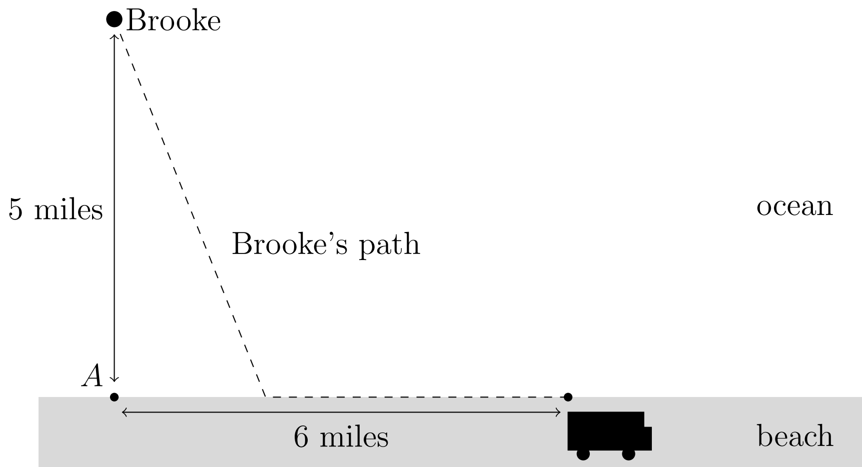
(c) Where is $p(t)$ increasing and where is it decreasing?

(d) What are the t-coordiates of the local minima and maxima?

(e) Using all the information above, and your answers to the questions, sketch a graph of $p(t)$ on the plane below.



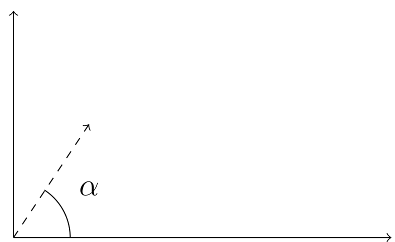
2. Brooke is located 5 miles out at sea from a straight shoreline in her kayak. She wants to make it to the taco truck on the beach for lunch, which is 6 miles from point A on shore (see picture). Brooke can paddle 2 miles/hour and walk 4 miles/hour. If she paddles along a straight line to shore, find an equation for the total time it will take Brooke to get to lunch. Your equation will depend on where Brooke beaches the boat. Where should she land the boat to eat as soon as possible?



Worksheet for Week 11: Maximizing functions

The best way to study for the Math 124 final is to solve challenging problems. In this worksheet, you'll solve a difficult problem that incorporates several topics from the class: position and velocity, parametric equations, derivatives, and maximizing functions by checking for critical points and also checking the endpoints.

You're on the planet Zed, which has no air and has gravitational constant $-g$. (Earth's gravitational constant is about $-9.8m/s^2$.) You throw a ball at speed v from the origin on a coordinate plane, where the horizontal axis lies along the ground of Zed. (See the picture.) The angle α is the angle above the horizontal that the ball is thrown. Suppose $0 \leq \alpha \leq \pi/2$.



Let $x(t)$, $y(t)$ denote the position of the ball at time t, where $t = 0$ is the instant the ball was thrown in the air. Then $x'(t)$, $y'(t)$ denote the horizontal and vertical velocities as functions of t.

In this situation (airless planet with gravitational constant $-g$), we have the formulas

$$y(t) = -\frac{1}{2}gt^2 + y'(0)t + y(0),$$
$$x(t) = x'(0)t + x(0).$$

1. What are $x'(0)$, $y'(0)$, $x(0)$ and $y(0)$? Use these numbers and the formulas above to find equations for $x'(t)$ and $y'(t)$.

2. When does the ball hit the ground?

3. When does the ball reach the peak of its trajectory?

4. How far from the origin is the ball when it hits the ground?

5. Which angle α will maximize the distance the ball travels?

6. In this problem, you'll figure out when the ball is farthest from the origin. The answer will depend on the angle α.

   (a) Find a formula $F(t)$ for the **square of** the distance from the ball to the origin at time t. (Maximizing the square of the distance is the same as maximizing the distance, and this way is less messy.)

(b) Find an expression for the non-zero critical numbers of $F(t)$.

(c) For some values of α, there are no non-zero critical numbers to check. For which α are there **no** non-zero roots of $F'(t)$?

(d) For the values of α you found in part (c), when is the ball the farthest from the origin?

(e) For which angle(s) α is there only one non-zero critical number of $F(t)$ to check?

(f) For the values of α you found in part (e), when is the ball the farthest from the origin? Be sure to check your answer.

(g) Are there any angles α so that the ball is farthest from the origin while it's in the air?