1. Compute the derivatives of the following functions. Do not simplify your answers.

(a) (4 points) $f(x) = \sqrt{\cos^2 x + 5x^7}$

$$f'(x) = \frac{-2\cos x \cdot \sin x + 35x^6}{2\sqrt{\cos^2 x + 5x^7}}$$

(b) (4 points) $g(t) = \tan^{-1}\left(\frac{5t+3}{t^2+4}\right)$

$$g'(t) = \frac{1}{1 + \left(\frac{5t+3}{t^2+4}\right)^2} \cdot \frac{5 \cdot (t^2+4) - 2t \cdot (5t+3)}{(t^2+4)^2}$$

(c) (5 points) $y = x^{\sqrt{x}}$

$$\begin{aligned}
\ln y &= \ln x^{\sqrt{x}} \\
&= \sqrt{x} \cdot \ln x \\
\frac{1}{y} \cdot \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \cdot \ln x + \sqrt{x} \cdot \frac{1}{x} \\
\frac{dy}{dx} &= y \cdot \left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right) \\
&= x^{\sqrt{x}} \cdot \left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right)
\end{aligned}$$

2. Consider the curve given by the parametric equations

$$\begin{aligned} x &= t^2 - 6t \\ y &= t - 3\ln t \end{aligned}$$

(a) (6 points) Find the equation of the tangent line to the curve when $t = 1$.

At $\quad t = 1$

$$\begin{aligned} x &= 1^2 - 6 \cdot 1 = -5 \\ y &= 1 - 3\ln 1 = 1 \\ \frac{dx}{dt} &= 2t - 6 \\ &= 2 \cdot 1 - 6 = -4 \\ \frac{dy}{dt} &= 1 - \frac{3}{t} \\ &= 1 - \frac{3}{1} = -2 \\ \frac{dy}{dx} &= \frac{dy/dt}{dx/dt} \\ &= \frac{-2}{-4} = \frac{1}{2} \end{aligned}$$

The equation of the tangent line is $\quad y - 1 = \frac{1}{2}(x + 5)$

(b) (6 points) Find all times $t \geq 0$ when the tangent line has slope equal to $\dfrac{1}{3}$.

$$\begin{aligned} \frac{1}{3} &= \frac{dy}{dx} \\ &= \frac{1 - \frac{3}{t}}{2t - 6} \\ 2t^2 - 6t &= 3t - 9 \\ 0 &= 2t^2 - 9t + 9 \\ &= (2t - 3)(t - 3) \\ t &= 3,\ \frac{3}{2} \end{aligned}$$

The derivative is undefined at $t = 3$. *The only correct value is* $\quad t = \dfrac{3}{2}$.

3. (8 points) Each side of a square is increasing at a rate of 2 feet/second. At what rate is the area of the square increasing when the area of the square is 49 square feet?

Let A be the area and x be the length of a side.

When $A = 49$, we have $x = 7$.

$$\begin{aligned} A &= x^2 \\ \frac{dA}{dt} &= 2x \cdot \frac{dx}{dt} \\ &= 2 \cdot 7 \cdot 2 \\ &= 28\, ft^2/sec \end{aligned}$$

4. (8 points) Find all the points (a,b) on the curve $x^2 + y^3 - 6x = 18$ where the tangent line is horizontal.

$$\begin{aligned} \frac{d}{dx}\left(x^2 + y^3 - 6x\right) &= \frac{d}{dx} 18 \\ 2x + 3y^2y' - 6 &= 0 \\ y' &= \frac{6-2x}{3y^2} \end{aligned}$$

$$\begin{aligned} \frac{6-2x}{3y^2} &= 0 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 3^2 + y^3 - 6 \cdot 3 &= 18 \\ y &= 3 \end{aligned}$$

The only point is $(3,3)$.

Note that the denominator in y' is not zero at this point.

5. (9 points) Let $x^2 - 6xy + y^3 = 8$. Find the value of y'' at the point where $x = 0$.

When $x = 0$, we get $y = 2$.

First compute y'.

$$\begin{aligned} \frac{d}{dx}\left(x^2 - 6xy + y^3\right) &= \frac{d}{dx}8 \\ 2x - 6y - 6xy' + 3y^2y' &= 0 \\ 2 \cdot 0 - 6 \cdot 2 - 6 \cdot 0 \cdot y' + 3 \cdot 2^2 \cdot y' &= 0 \\ -12 + 12y' &= 0 \\ y' &= 1 \end{aligned}$$

Now compute y''.

$$\begin{aligned} \frac{d}{dx}\left(2x - 6y - 6xy' + 3y^2y'\right) &= \frac{d}{dx}0 \\ 2 - 6y' - 6y' - 6xy'' + 6yy'y' + 3y^2y'' &= 0 \\ 2 - 6 \cdot 1 - 6 \cdot 1 - 6 \cdot 0 \cdot y'' + 6 \cdot 2 \cdot 1^2 + 3 \cdot 2^2 \cdot y'' &= 0 \\ 2 + 12y'' &= 0 \\ y'' &= -\frac{1}{6} \end{aligned}$$

This page is for extra work.