1. **[5 points per part]** For each of the following functions, compute $f'(x)$

(a) $f(x) = \ln(\sec(x) + e^x)$

$$f'(x) = \boxed{\frac{\sec(x)\tan(x) + e^x}{\sec(x) + e^x}}$$

deriv of $\sec(x)+e^x$

(b) $f(x) = \arcsin(3x^2)$

$$f'(x) = \frac{1}{\sqrt{1-(3x^2)^2}} \cdot 6x = \boxed{\frac{6x}{\sqrt{1-9x^4}}}$$

(c) $f(x) = (\cos(x) + 2)^{\sqrt{x}}$

$$y = (\cos(x)+2)^{\sqrt{x}}$$

$$\ln(y) = \ln\left((\cos(x)+2)^{\sqrt{x}}\right) = \sqrt{x}\,\ln(\cos(x)+2)$$

$$\downarrow \frac{d}{dx}$$

$$\frac{y'}{y} = \frac{1}{2\sqrt{x}}\ln(\cos(x)+2) + \sqrt{x}\,\frac{\sin(x)}{\cos(x)+2}$$

$$\boxed{y' = (\cos(x)+2)^{\sqrt{x}}\left(\frac{\ln(\cos(x)+2)}{2\sqrt{x}} + \frac{\sqrt{x}\sin(x)}{\cos(x)+2}\right)}$$

2. **[12 points]** Consider the following parametric curve on the domain $t > 0$:

$$x(t) = t^2 - 12t + 10\ln(t) \qquad y(t) = 5\arctan(t) - t$$

Find all points on the curve where the tangent line is **vertical**, and all points where it is **horizontal**. (Specify which is which.)

Write your answers in exact form. You do not need to simplify.

$y'(t) = 0,\ x'(t) \neq 0$

$x'(t) = 0,\ y'(t) \neq 0$

$$x'(t) = 2t - 12 + \frac{10}{t} = 0$$

$$2t^2 - 12t + 10 = 0$$

$$t^2 - 6t + 5 = 0$$

$$(t-1)(t-5) = 0$$

$$t = 1 \text{ or } 5$$

$$y'(t) = \frac{5}{1+t^2} - 1 = 0$$

$$\frac{5}{1+t^2} = 1$$

$$5 = 1 + t^2$$

$$t^2 = 4$$

$t = 2$ (not -2 b/c of domain)

Horizontal at $t = 2$

$$\boxed{\left(-20 + 10\ln(2),\ 5\arctan(2) - 2\right)}$$

Vertical at $t = 1$ & $t = 5$

$$\boxed{\left(-11,\ \frac{5\pi}{4} - 1\right) \ \&\ \left(-35 + 10\ln(5),\ 5\arctan(5) - 5\right)}$$

3. **[9 points]** Let $f(x) = x\sqrt{4x+1}$.

Use the linearization of f at $x = 2$ to find an approximate solution to the equation

$$x\sqrt{4x+1} = 5.935.$$

$$f'(x) = \sqrt{4x+1} + \frac{4x}{2\sqrt{4x+1}}$$

$$f'(2) = 3 + \frac{8}{6} = \frac{13}{3} \qquad f(2) = 6$$

$$L(x) = 6 + \frac{13}{3}(x-2)$$

$$5.935 = 6 + \frac{13}{3}(x-2)$$

$$-0.065 = \frac{13}{3}(x-2)$$

$$-0.015 = x - 2$$

$$\boxed{x = 1.985}$$

4. **[9 points]** Consider the curve defined implicitly by the equation $\cos(\pi x) + 3^y = x^2 y + 6$.

Find the equation of the line tangent to this curve at the point $(1, 2)$. $\frac{d}{dx}$

$$-\pi\sin(\pi x) + 3^y \ln(3)\frac{dy}{dx} = 2xy + x^2\frac{dy}{dx}$$

$$3^y \ln(3)\frac{dy}{dx} - x^2\frac{dy}{dx} = 2xy + \pi\sin(\pi x)$$

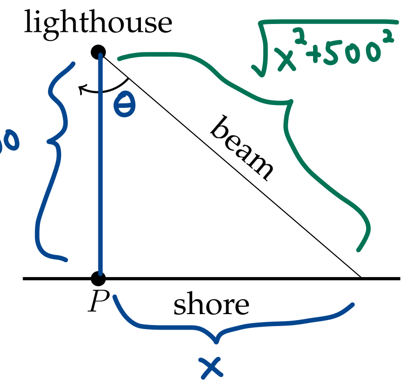
$$\frac{dy}{dx} = \frac{2xy + \pi\sin(\pi x)}{3^y\ln(3) - x^2} \xrightarrow[y=2]{x=1} \frac{4}{9\ln(3) - 1}$$

$$\boxed{y = \frac{4}{9\ln(3)-1}(x-1) + 2}$$

5. **[15 points]** A lighthouse is located on an island 500 meters from the nearest point P on a straight shoreline. Its beam rotates at a constant speed.

When the beam hits a point on the shore 100 meters from P, it's moving at 150 meters per second along the shoreline.

How long does it take the beam to make one complete revolution?



$$\tan(\theta) = \frac{x}{500}$$

$$\downarrow \frac{d}{dt}$$

$$\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{500}\frac{dx}{dt}$$

$$\frac{x^2+500^2}{500^{\cancel{2}}}\frac{d\theta}{dt} = \frac{\frac{dx}{dt}}{\cancel{500}} \qquad \sec(\theta) = \frac{\sqrt{x^2+500^2}}{500}$$

Plug in $x=100$, $\frac{dx}{dt} = -150$

$$\frac{100^2+500^2}{500}\frac{d\theta}{dt} = -150$$

$$\frac{d\theta}{dt} = \frac{-75000}{260000} = \frac{-15}{52} \text{ rad/sec}$$

So it takes $\dfrac{2\pi}{\left(\frac{15}{52}\right)} = \dfrac{104\pi}{15} \approx \boxed{21.78 \text{ seconds}}$

to make one revolution