Name: ________________

TA/section: ________________

Instructions: Unless otherwise stated, you MUST show work for credit. No notes allowed. A scientific calculator is allowed. No graphing calculators. Unless instructed otherwise, you can leave your answers in either exact form or use three decimal places of accuracy.

SCORING:

1. _____/12
2. _____/9
3. _____/10
4. _____/8
5. _____/11

/ 50
1. (12 points; 3pts each) Using the **derivative rules** you have learned, compute the
derivatives. You do not need to simplify your final answer. You must BOX YOUR
FINAL ANSWER.

(a) \( y = \sin^{-1}(\ln(x^2 + 1)) \),

\[ y' = \]

(b) \( y = \sin^2(3t^2 - t + 1) \),

\[ \frac{dy}{dt} = \]
1. (cont.)

(c) If \( f(x) = \sqrt{x + \sqrt{2x}} \),

\[ f'(x) = \]

(d) \( y = x^{\ln(x)} \),

\[ y'(x) = \]
2. (9 points) The graphs of \( f(x) = x^2 + 1 \) and \( g(x) = -x^2 + x \) are pictured below. Find the equation of the pictured line which is simultaneously tangent to both curves.
3. (10 points) The graph of the equation \(2(x^2 + y^2)^2 = 25(x^2 - y^2)\) is pictured below.

(a) (6pts) Find the equation of the tangent line to the curve at \((3, 1)\).

(b) (4pts) Let \(Q = (a, 1.01)\) be the point on the curve in the first quadrant with \(y\)-coordinate 1.01. Using linear approximation, estimate the value of \(a\). Leave your answer in exact form.
4. (8 points) Sand is being dumped from a conveyor belt at a rate of $2 \text{ m}^3/\text{min}$ and forms a right circular cone. Assume the radius of the cone is always three times as large as the height of the cone. (Recall, the volume of a right circular cone is $V = \frac{1}{3}\pi r^2h$, where $r$ is the radius of the circular base and $h$ is the height of the cone.) Find the rate at which the height of the cone is increasing when the height is $9\text{m}$. 
5. (11 points) A particle is moving in the \(xy\)-plane with parametric equations

\[
x(t) = e^t + e^{-t} \\
y(t) = e^{-t}
\]

at time \(t\) seconds, \(t \geq 0\). The units on each axis are centimeters (cm). Recall that the speed of the particle is given by the formula

\[
s(t) = \sqrt{(x'(t))^2 + (y'(t))^2}.
\]

This problem studies the values of the speed on the time interval \([0,1]\).

(a) (3pts) Find \(s(t)\) as an explicit function of \(t\).

(b) (2pts) Calculate \(s(0)\) and \(s(1)\).

(c) (6pts) Find the critical numbers for \(s(t)\) and the minimum speed on the time interval \([0,1]\).