Math 124 B Winter 2025 Midterm 2

Print Your Full Name: Solutions

Signature:

Student ID Number:

Quiz Section:

Instructor's Name:

TA's Name:

Please read these instructions!

1. Your exam contains 8 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Unless otherwise stated, make sure to SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a $\boxed{\text{box around your final answer}}$ to each question.
6. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π, $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course or from other courses.

Problem	Total Points
1	10
2	10
3	10
4	10
5	10
Total	50

1. Find the derivative of the following functions using derivative rules. Do not simplify your final answer.

(a) (3 points) $y = \arctan^2(\sin(x))$ $= (\arctan(\sin x))^2$

$$\boxed{y' = \underbrace{2(\arctan(\sin(x)))}_{+1} \cdot \underbrace{\frac{1}{1+(\sin x)^2}}_{+1} \cdot \underbrace{\cos x}_{+1}}$$

(b) (3 points) $y = \sqrt{2x + \sqrt{4x^2 - x}}$

$$\boxed{y' = \underbrace{\frac{1}{2\sqrt{2x+\sqrt{4x^2-x}}}}_{+1}\left(\underbrace{2 + \frac{1}{2\sqrt{4x^2-x}}}_{+1}\underbrace{(8x-1)}_{+1}\right)}$$

(c) (4 points) $y = x^{(x^3)}$

$$\ln y = \ln\left(x^{x^3}\right)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(x^3 \cdot \ln(x)\right) \quad +1$$

$$\frac{1}{y} \cdot y' = x^3 \cdot \frac{1}{x} + \ln(x) \cdot 3x^2$$

$$\boxed{y' = \underbrace{x^{x^3}}_{+1}\underbrace{\left(x^2 + 3x^2 \cdot \ln(x)\right)}_{+2}}$$

2. (10 points) If $2xy + 8e^y = 8e$, find y'' when $x = 0$.

$x = 0$

$\rightarrow 2(0)y + 8e^y = 8e$

$8e^y = 8e$

$y = 1$ (+1)

$$\frac{d}{dx}(2xy + 8e^y) = \frac{d}{dx}(8e) \quad (+1)$$

$$\left[2x \cdot \frac{dy}{dx} + y \cdot 2 + 8e^y \cdot \frac{dy}{dx} = 0\right] \quad (+2)$$

$$\frac{dy}{dx}(2x + 8e^y) = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{2x + 8e^y} = \frac{-y}{x + 4e^y} \quad (+1)$$

$$\frac{d^2y}{dx^2} = \frac{(x+4e^y) \cdot -\frac{dy}{dx} - (-y) \cdot \left(1 + 4e^y \cdot \frac{dy}{dx}\right)}{(x+4e^y)^2} \quad (+3)$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=0 \\ y=1}} = \frac{-1}{4e} \quad (+1)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\overbrace{(0+4e) \cdot \frac{1}{4e}}^{=1} + \overbrace{1\left(1 + 4e \cdot -\frac{1}{4e}\right)}^{=0}}{(4e)^2}$$

$$= \boxed{\frac{1}{(4e)^2}} \quad (+1)$$

3. (10 points) Find an equation of the tangent line to the parametric curve

$$x = t^2\cos(t), \quad y = t^2\sin(t)$$

at the point corresponding to $t = \pi/2$. Expressions involving trigonometric functions must be fully simplified to their exact form.

$$\frac{dx}{dt} = t^2 \cdot -\sin t + \cos t \cdot 2t \quad +3$$

$$\frac{dy}{dt} = t^2 \cdot \cos t + \sin t \cdot 2t \quad +3$$

$$\Rightarrow \frac{dy}{dx} = \frac{t^2 \cdot \cos t + \sin t \cdot 2t}{-t^2 \sin t + \cos t \cdot 2t} \quad +1$$

$$t = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = \frac{0 + 1 \cdot 2 \cdot \frac{\pi}{2}}{-\left(\frac{\pi}{2}\right)^2 \cdot 1 + 0} = \frac{\pi}{-\frac{\pi^2}{4}} = \frac{-4}{\pi} \quad +1$$

Point: $x = \left(\frac{\pi}{2}\right)^2 \cdot \cos\frac{\pi}{2} = 0$

$y = \left(\frac{\pi}{2}\right)^2 \cdot \sin\frac{\pi}{2} = \frac{\pi^2}{4}$ +1

Eq. of tan. line: $\boxed{y = -\frac{4}{\pi}(x - 0) + \frac{\pi^2}{4}}$ +1

4. (10 points) Use a linear approximation to $f(x) = \sqrt[3]{x}$ to estimate $\sqrt[3]{27.1}$. Round your number to three decimal places.

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$, $a = 27$ +2 $\quad f(27) = \sqrt[3]{27} = 3$ +1

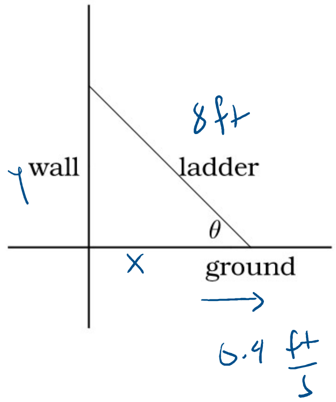
$f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$ +3 $\quad f'(27) = \frac{1}{3}(27)^{-\frac{2}{3}} = \frac{1}{27}$ +1

$\Rightarrow L(x) = \frac{1}{27}(x - 27) + 3$ +2

$\Rightarrow f(27.1) \approx L(27.1) = \frac{1}{27}(27.1 - 27) + 3$

$= \frac{1}{27} \cdot \frac{1}{10} + 3 \approx \boxed{3.004}$ +1

5. (10 points) A ladder 8 ft long leans against a vertical wall (see the figure below). Assume the bottom of the ladder slides away from the wall at a rate of 0.4 ft/s. How fast is the angle θ between the ladder and the ground changing when the top of the ladder is 4 ft from the ground?

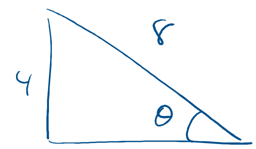


Given

$$\frac{dx}{dt} = 0.4\ \frac{ft}{s}$$

Wanted

$$\frac{d\theta}{dt} \text{ when } y = 4\ ft$$



$$\sin\theta = \frac{4}{8}$$

$$\sin\theta = \frac{1}{2} \quad +1$$

$$\cos\theta = \frac{x}{8} \quad +4$$

$$\frac{d}{dt}\left(8\cos\theta\right) = \frac{d}{dt}(x) \quad +1$$

$$8 \cdot -\sin\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt} \quad +1$$

$$+2 \quad \frac{d\theta}{dt} = -\frac{1}{8\sin\theta} \cdot \frac{dx}{dt}$$

$$\Rightarrow \quad \frac{d\theta}{dt} = -\frac{1}{8 \cdot \frac{1}{2}} \cdot 0.4$$

$$= -\frac{1}{4} \cdot 0.4 = \boxed{-\frac{1}{10}\ \text{rad/sec}} \quad +1$$

Extra scratch paper.

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