Math 124 B and D Midterm 2 Solutions

Print Your Full Name	Signature
Solutions	
Student ID Number	**Quiz Section**
Instructor's Name	**TA's Name**

Please read these instructions!

1. Your midterm exam contains 7 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The entire midterm exam is worth 42 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Make sure to ALWAYS SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Partial credit is awarded as earned. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a $\boxed{\text{box around your final answer}}$ to each question. Do not box in multiple, different final answers, as this may result in loss of credit.
6. There is plenty of space on the exam to do your work. If you need extra space, use the last two pages of the exam. Clearly indicate that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π, $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course.

Problem	Total Points	Score
1	12	
2	10	
3	10	
4	10	
Total	42	

1. Find the derivatives of the following functions. Do not simplify your answers.

(a) (3 points) $y = \ln(\cos^2(x))$

$$\boxed{y' = \frac{1}{\cos^2 x} \cdot 2\cos(x) \cdot -\sin x}$$

(b) (3 points) $y = 5^{x^4}$

$$\boxed{y' = 5^{x^4} \cdot \ln 5 \cdot 4x^3}$$

(c) (3 points) $y = \arctan\left(\frac{1}{x}\right)$

$$\boxed{y' = \frac{1}{1 + \left(\frac{1}{x}\right)^2} \cdot -\frac{1}{x^2}}$$

(d) (3 points) $y = (\sin x)^{x^2}$

$$\ln y = \ln\left((\sin x)^{x^2}\right)$$

$$\ln y = x^2 \cdot \ln(\sin x)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(x^2 \cdot \ln(\sin x)\right)$$

$$\frac{1}{y} \cdot y' = x^2 \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot 2x$$

$$\boxed{y' = (\sin x)^{x^2}\left(\frac{x^2 \cdot \cos x}{\sin x} + \ln(\sin x) \cdot 2x\right)}$$

Alternate solution

$$y = e^{\ln((\sin x)^{x^2})}$$

$$y = e^{x^2 \cdot \ln(\sin x)}$$

$$y' = e^{x^2 \cdot \ln(\sin x)} \cdot \left(x^2 \cdot \frac{1}{\sin x} \cos x + \ln(\sin x) \cdot x^2\right)$$

$$= (\sin x)^2\left(\frac{x^2 \cos x}{\sin x} + \ln(\sin x) \cdot x^2\right)$$

2. The following parts (a) and (b) of this problem are not related.

(a) (5 points) Find y' if $\sin(6xy) = x\cos(4y)$. Then find the equation of the tangent line to the curve at the point $(1, \pi/4)$.

$$\cos(6xy)\cdot(6x\cdot y' + y\cdot 6) = x\cdot -\sin(4y)\cdot 4y' + \cos(4y)$$

$$y'\left(\cos(6xy)\cdot 6x + 4x\sin(4y)\right) = \cos(4y) - 6y\cos(6xy)$$

$$y' = \frac{\cos 4y - 6y\cdot\cos(6xy)}{\cos(6xy)\cdot 6x + 4x\sin(4y)}$$

$$\text{At } \left(1, \tfrac{\pi}{4}\right) \Rightarrow y' = \frac{-1 - \frac{6\pi}{4}\cdot\cos\left(\frac{6\pi}{4}\right)}{\underbrace{\cos\left(\frac{6\pi}{4}\right)\cdot 6}_{=0} + \underbrace{4\cdot 1\cdot\sin(\pi)}_{=0}}$$

undefined slope means vertical tangent line

$$\boxed{\Rightarrow \text{Equation of tangent line is } x = 1.}$$

(b) (5 points) Find y'' if $3x + 2y^2 - xy = 1$. Do not simplify your answer.

$$3 + 4y\cdot y' - (xy' + y) = 0$$

$$y'(4y - x) = -3 + y \longrightarrow y' = \frac{y-3}{4y-x}$$

$$y'' = \frac{(4y-x)\cdot(y') - (y-3)\cdot(4y'-1)}{(4y-x)^2}$$

$$= \boxed{\frac{(4y-x)\cdot\left(\frac{y-3}{4y-x}\right) - (y-3)\left(4\left(\frac{y-3}{4y-x}\right)-1\right)}{(4y-x)^2}}$$

3. (10 points) Consider the curve in the xy-plane with parametric equations

$$x = \sqrt{t} + t, \quad y = t + \sqrt{2}\sin(t), \quad 0 < t < 2\pi.$$

Find the points on the parametric curve where the tangent line is horizontal. Make sure your final answer has fully simplified all expressions involving trigonometric functions.

Need points where $\frac{dy}{dx} = 0.$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 + \sqrt{2}\cos t}{\frac{1}{2\sqrt{t}} + 1} = 0 \Rightarrow 1 + \sqrt{2}\cos t = 0$$

$$\cos t = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow t = \frac{3\pi}{4}, \frac{5\pi}{4}.$$

- $t = \frac{3\pi}{4}$

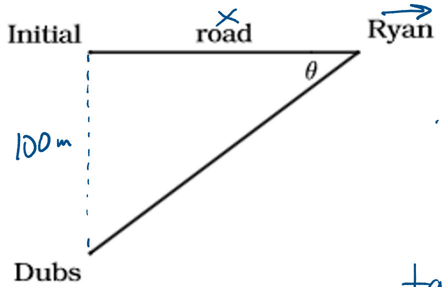
$$\Rightarrow x = \sqrt{\frac{3\pi}{4}} + \frac{3\pi}{4}, \quad y = \frac{3\pi}{4} + \sqrt{2}\cdot\sin\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} + \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \frac{3\pi}{4} + 1$$

- $t = \frac{5\pi}{4}$

$$\Rightarrow x = \sqrt{\frac{5\pi}{4}} + \frac{5\pi}{4}, \quad y = \frac{5\pi}{4} + \sqrt{2}\cdot\sin\left(\frac{5\pi}{4}\right) = \frac{5\pi}{4} - \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \frac{5\pi}{4} - 1$$

$$\boxed{\text{Points are } \left(\sqrt{\frac{3\pi}{4}} + \frac{3\pi}{4}, \frac{3\pi}{4} + 1\right), \left(\sqrt{\frac{5\pi}{4}} + \frac{5\pi}{4}, \frac{5\pi}{4} - 1\right)}$$

4. (10 points) On a drive through the UW Seattle campus, Ryan is taking photographs of Dubs. Suppose Ryan's car travels east at 10 meters per second along a straight path, and initially, Dubs is 100 meters directly south of the car (see image and compass below). After 12 seconds from the initial position, what is the rate of change of the angle θ made between the road and the straight line from Ryan's car to Dubs? Write your answer as a simplified fraction or an approximation to two decimal places, including units.



Given

$\frac{dx}{dt} = 10 \frac{m}{s}$

Wanted

$\frac{d\theta}{dt}$ when $t = 12 \sec$

$$\tan\theta = \frac{100}{x}$$

$$\theta = \tan^{-1}\left(\frac{100}{x}\right)$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{100}{x}\right)^2} \cdot \frac{-100}{x^2} \cdot \frac{dx}{dt}$$

Ryan's velocity is $10 \frac{m}{s} \Rightarrow x = 10 \frac{m}{s} \cdot 12s = 120m$ when $t = 12 \sec$

$$\therefore \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{100}{120}\right)^2} \cdot -\frac{100}{(120)^2} \cdot 10$$

$$= \frac{-1000}{120^2 + 100^2} = \frac{-1000}{24400} = \frac{-10}{244} = \boxed{\frac{-5}{122} \frac{\text{rad}}{\sec} \approx -0.04 \frac{\text{rad}}{\sec}}$$

Extra scratch paper.

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