Solutions to Math 124 C Spring 2023 Midterm II

1. (a) $y' = \dfrac{7\sin^6(e^x)\cdot\cos(e^x)\cdot e^x}{2\sqrt{1+\sin^7(e^x)}}$

   (b)
$$\sec^2(x+y)\cdot(1+y') - 12x^2y^2 - 8x^3yy' = e^{xy}(y+xy')$$
$$\left(\sec^2(x+y) - 8x^3yy' - xe^{xy}\right)y' = -\sec^2(x+y) + 12x^2y^2 + ye^{xy}$$
$$y' = \frac{-\sec^2(x+y) + 12x^2y^2 + ye^{xy}}{\sec^2(x+y) - 8x^3yy' - xe^{xy}}$$

   (c)
$$\ln y = 5x\cdot\ln\left(1+x^2\right)$$
$$\frac{y'}{y} = 5\ln\left(1+x^2\right) + 5x\cdot\frac{2x}{1+x^2}$$
$$y' = \left(5\ln\left(1+x^2\right) + \frac{10x^2}{1+x^2}\right)\left(1+x^2\right)^{5x}$$

2. (a) We evaluate the derivative
$$\frac{dy}{dx} = \frac{3t^2-12}{4t}$$
at the t value where $(2t^2-40, t^3-12t) = (-38,-11)$. Since the x coordinate is a quadratic, it is easier to solve
$$-38 = 2t^2 - 40$$
to get $t \pm 1$. Only $t=1$ works for $t^3-12t=-11$ so $t=1$. The slope of the tangent line is
$$\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{3t^2-12}{4t}\right|_{t=1} = -\frac{9}{4}$$
so the equation of the tangent line is
$$y + 11 = -\frac{9}{4}(x+38)$$
or $y = -\dfrac{9}{4}x - \dfrac{298}{4}$.

   (b) Intersection the tangent line with the corve
$$t^3 - 12t + 11 = \frac{9}{4}(2t^2 - 40 + 38)$$
which simplifies to
$$4t^3 + 18t^2 - 48t + 26 = 0$$
Since $t=1$ is a root we get
$$(t-1)(4t^2+22t-26) = 0$$
In fact, $t=1$ is a double root
$$(t-1)^2(4t+26) = 0$$
so the second point of intersection is when $t = -13/2$ and the point is $\left(\dfrac{89}{2}, -\dfrac{1573}{8}\right) = (44.5, -196.625)$

3. First to write the tangent line at $(1, P(1)) = (1, -4)$ we compute

$$p'(x) = 12x^2 - 2x + 3.$$

So the slope is $p'(1) = 13$ and the tangent line is given by

$$y + 1 = 13(x - 1)$$

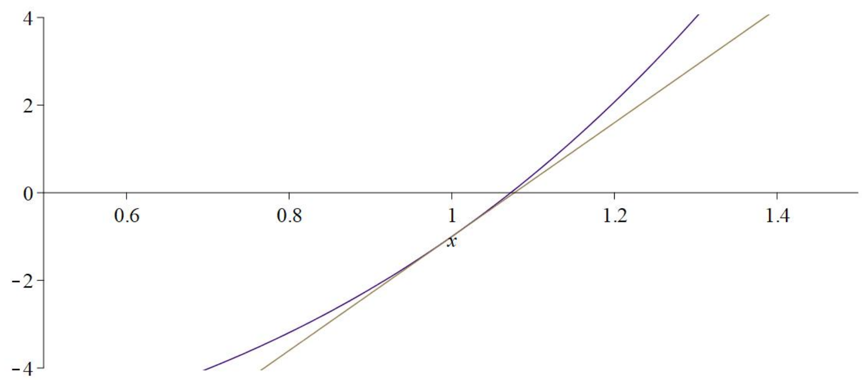
Now we use approximation:

$$0 + 1 \approx 13(x - 1)$$

to get $x \approx 14/13$. To see if it is more or less, we look at the second derivative

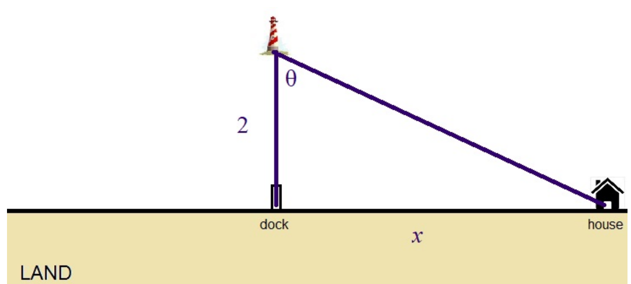
$$p''(x) = 24x - 2x$$

and $p''(1) - 22 > 0$. So, near the point $(1, -1)$ we have a concave up increasing picture:



where the gold tangent stays below and to the right of the purple curve so the approximation is more than the actual value.

4. Given $\dfrac{dx}{dt} = -4$ meters per second, to find $\dfrac{d\theta}{dt}$



we use the equation

$$\tan\theta = \frac{x}{2000}.$$

The derivative with respect to t is

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{2000} \cdot \frac{dx}{dt}.$$

When $x = 3$ kilometers, the hypotenuse would be $\sqrt{13}$ so $\sec\theta = \frac{\sqrt{13}}{2}$. Then,

$$\left(\frac{\sqrt{13}}{2}\right)^2 \frac{d\theta}{dt} = \frac{1}{2000} \cdot (-4).$$

so $\frac{d\theta}{dt} = \frac{1}{1625}$ radians per second or $\frac{60}{3250\pi}$ revolutions per minute.