

Solutions to Math 124 C Spring 2023 Midterm II

1. (a) $y' = \frac{7 \sin^6(e^x) \cdot \cos(e^x) \cdot e^x}{2\sqrt{1 + \sin^7(e^x)}}$

(b)

$$\sec^2(x + y) \cdot (1 + y') - 12x^2y^2 - 8x^3yy' = e^{xy}(y + xy')$$

$$(\sec^2(x + y) - 8x^3yy' - xe^{xy})y' = -\sec^2(x + y) + 12x^2y^2 + ye^{xy}$$

$$y' = \frac{-\sec^2(x + y) + 12x^2y^2 + ye^{xy}}{\sec^2(x + y) - 8x^3yy' - xe^{xy}}$$

(c)

$$\ln y = 5x \cdot \ln(1 + x^2)$$

$$\frac{y'}{y} = 5 \ln(1 + x^2) + 5x \cdot \frac{2x}{1 + x^2}$$

$$y' = \left(5 \ln(1 + x^2) + \frac{10x^2}{1 + x^2}\right) (1 + x^2)^{5x}$$

2. (a) We evaluate the derivative

$$\frac{dy}{dx} = \frac{3t^2 - 12}{4t}$$

at the t value where $(2t^2 - 40, t^3 - 12t) = (-38, -11)$. Since the x coordinate is a quadratic, it is easier to solve

$$-38 = 2t^2 - 40$$

to get $t \pm 1$. Only $t = 1$ works for $t^3 - 12t = -11$ so $t = 1$. The slope of the tangent line is

$$\left. \frac{dy}{dx} \right|_{t=1} = \left. \frac{3t^2 - 12}{4t} \right|_{t=1} = -\frac{9}{4}$$

so the equation of the tangent line is

$$y + 11 = -\frac{9}{4}(x + 38)$$

or $y = -\frac{9}{4}x - \frac{193}{2}$.

(b) Intersection the tangent line with the curve

$$t^3 - 12t + 11 = \frac{9}{4}(2t^2 - 40 + 38)$$

which simplifies to

$$4t^3 + 18t^2 - 48t + 26 = 0$$

Since $t = 1$ is a root we get

$$(t - 1)(4t^2 + 22t - 26) = 0$$

In fact, $t = 1$ is a double root

$$(t - 1)^2(4t + 26) = 0$$

so the second point of intersection is when $t = -13/2$ and the point is $\left(\frac{89}{2}, -\frac{1573}{8}\right) = (44.5, -196.625)$

3. First to write the tangent line at $(1, P(1)) = (1, -4)$ we compute

$$p'(x) = 12x^2 - 2x + 3.$$

So the slope is $p'(1) = 13$ and the tangent line is given by

$$y + 1 = 13(x - 1)$$

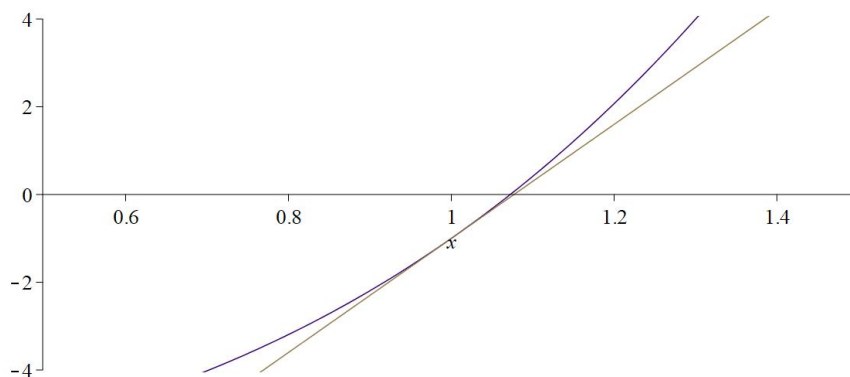
Now we use approximation:

$$0 + 1 \approx 13(x - 1)$$

to get $x \approx 14/13$. To see if it is more or less, we look at the second derivative

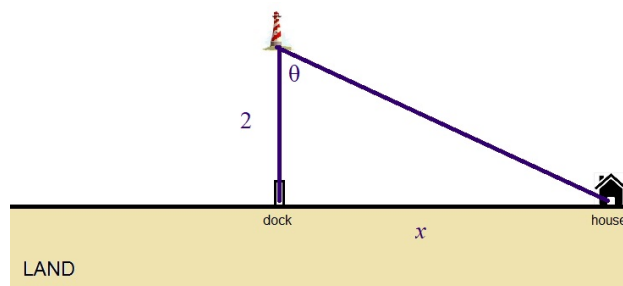
$$p''(x) = 24x - 2$$

and $p''(1) = 22 > 0$. So, near the point $(1, -1)$ we have a concave up increasing picture:



where the gold tangent stays below and to the right of the purple curve so the approximation is more than the actual value.

4. Given $\frac{dx}{dt} = -4$ meters per second, to find $\frac{d\theta}{dt}$



we use the equation

$$\tan \theta = \frac{x}{2000}.$$

The derivative with respect to t is

$$\sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{1}{2000} \cdot \frac{dx}{dt}.$$

When $x = 3$ kilometers, the hypotenuse would be $\sqrt{13}$ so $\sec \theta = \frac{\sqrt{13}}{2}$. Then,

$$\left(\frac{\sqrt{13}}{2}\right)^2 \frac{d\theta}{dt} = \frac{1}{2000} \cdot (-4).$$

so $\frac{d\theta}{dt} = \frac{1}{1625}$ radians per second or $\frac{60}{3250\pi}$ revolutions per minute.