Math 124 G Fall 2025 Midterm II

November 18, 2025

Name_____

Student Number_____

Instructions

- These exams will be scanned. **Please write your name and student number clearly.**

- There are 4 questions. The exam is out of 50 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. **Hand in your notes with your exam.**

- You can only use a Ti-30x IIS calculator. **You have to give exact answers to questions on this exam.** ($\frac{2\ln 3}{\pi}$ and 1/3 are exact, 0.699 and 0.333 are approximations for those numbers.)

- Show your work. If we cannot read or follow your work, we cannot grade it. If you have read all the instructions carefully, put a smiley next to your student number for a bonus point. **You may not get full credit for a right answer if your answer is not justified by your work.**

1. (12 points) Compute $\dfrac{dy}{dx}$ for the following.

(a) $y = \sqrt{9x + \sqrt{8x + 7e^x}}$

(b) $y = e^{\tan x} + \tan(e^x) + (\tan x)^e$

(c) $y = (1 + 2x^2)^{(3x+4)}$

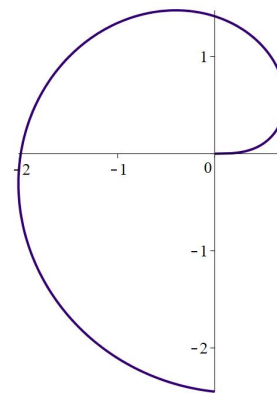2. (13 points) A curve is given by the implicit equation

$$2y^2x^3 + \ln\left(\frac{2x}{y}\right) = 8$$

(a) Use linear approximation near $(1, 2)$ to approximate the value of y when $x = 1.05$.

(b) Compute the value of y'' at the point $(1, 2)$ to decide whether your answer in part(a) is less than or more than the actual value of y.

3. (15 points) A particle is moving along the parametric curve

$$x = \sqrt{t}\cos\left(\frac{\pi}{4}t\right) \qquad y = \sqrt{t}\sin\left(\frac{\pi}{4}t\right)$$

where t is in seconds and the axes have units centimeters. It traces the spiral shown on the right.

(a) What is the speed of the particle at $t = 2$?

(b) How fast is the particle's distance from the origin changing at $t = 3$?

(c) At $t = 4$ the magnetic field which has kept this particle on the spiral is turned off and the particle flies off in a path tangent to the curve. Find the equation of this tangent path.

4. (10 points) You are preparing pour-over coffee before you start studying for your math final. The cone with the filter has radius 5 centimeters and height 12 centimeters. Your favorite $\mathbb{W}$ mug has the shape of a cylinder and has radius 3 centimeters and height 7 centimeters. The picture is not to scale.

Staring at the coffee being brewed, you see that the depth of the coffee in the filter is falling at a rate of 1.5 centimeters per minute the moment it is 6 centimeters deep at its deepest point measured at the center of the cone.

Keeping in mind that all the coffee that comes out of the cone ends up in the cup, how fast is the height of the coffee in the cup rising at that moment?

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