Math 124D – Midterm II

Autumn 2021

Time: 80 mins.

1. Answer all questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. There are also two blank pages at the end for scratch work or continued answers.
2. Unless stated otherwise, justify your answers to receive full credit. Your answers do not have to be in complete sentences, but they do need to be understandable.
3. You can use a TI-30X IIS calculator. No other calculator is allowed.
4. You can use a page of notes during the midterm. It must be single sided on standard letter sized paper (8.5 by 11 inches), handwritten, and not an exact copy of another students page of notes. You should write your name on the sheet and turn it in with your exam.

Name: _____

Student ID #: _____

Question	Points	Score
1	12	
2	12	
3	10	
4	12	
5	4	
Total:	50	

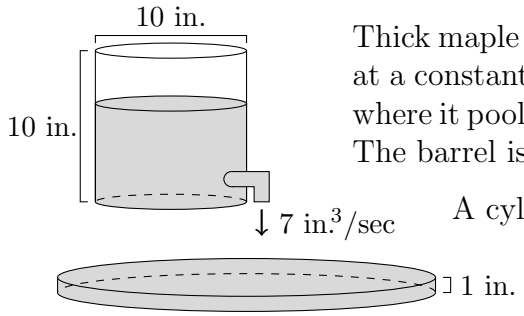
1. (12 points) Find the derivatives $y'(x) = \dfrac{dy}{dx}$ of the following functions.
   (Your final answers should be in terms of x only.)

   (a) $y = \sqrt{x \sin(5x)}$

   (b) $y = \arctan\left(\dfrac{x^3}{x+1}\right)$ (Here $\arctan(x)$ denotes the inverse function of $\tan(x)$.)

   (c) $y = x^{(e^x)}$

2. (12 points)



Thick maple syrup starts oozing out of a full cylindrical barrel at a constant rate of 7 cubic inches per second onto the floor, where it pools in cylindrical shape of uniform 1-inch thickness. The barrel is 10 inches in both height and diameter.

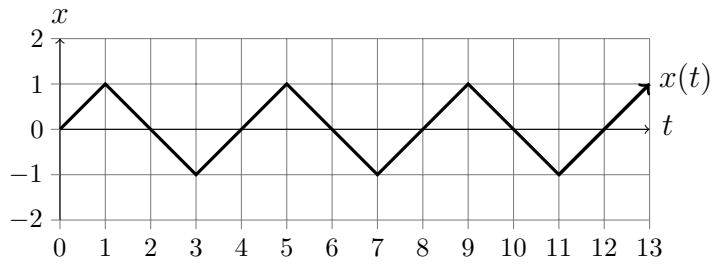
A cylinder of radius r and height h has volume $\pi r^2 h$.

Include units in your answers!

(a) What is the rate of change of the height of syrup in the barrel when the syrup level is 4 inches from the top of the barrel?

(b) At what rate is the diameter of the maple syrup on the floor changing at this time?

3. (10 points) Consider the curve parametrized by $(x(t), y(t))$ where $x(t)$ is graphed below and $y(t) = \cos(\pi t/2)$.



(a) Find a formula for $y'(x) = \dfrac{dy}{dx}$ when $1 < t < 3$.

(b) Find $y''(x) = \dfrac{d^2y}{dx^2}$ when $t = 2$.

(c) What is the smallest positive value of t for which the point $(x(t), y(t))$ returns to its starting position at $t = 0$?

4. (12 points) Consider the implicit equation $y^7 - y = x$.

   (a) Find a formula for $y'(x) = \dfrac{dy}{dx}$ in terms of x and y.

   (b) Find a formula for $y''(x) = \dfrac{d^2y}{dx^2}$ in terms of x and y.

   (c) Use a linear approximation of the curve at $(0, 1)$ to estimate a value of y so that the point $(\frac{1}{2}, y)$ lies on the curve.

   (d) Does your answer from part (c) over- or under-estimate the true value?

5. (4 points) From the definition of the derivative, the following limit can be written as $f'(a)$ for some function $f(x)$ and value $x = a$:

$$\lim_{h\to 0} \ln((1+h)^{1/h}) = \lim_{h\to 0} \frac{\ln(1+h)}{h} = \lim_{h\to 0} \frac{\ln(1+h) - \ln(1)}{h}.$$

(a) What is a choice of $f(x)$ and a so that this limit equals $f'(a)$?

(b) Use rules of differentiation to find $f'(x)$ and evaluate to find the above limit.

(c) Use $e^{\ln(x)} = x$ and your answer in part (b) to find $\lim_{h\to 0}(1+h)^{1/h}$.
(Keep your answer in exact form, not numerical.)