

Name _____

Math 124

Second Midterm

8:30-9:50, Feb. 21, 2017

(80 minutes — 100 points)

DIRECTIONS: No calculator or other electronic device is permitted. You may have 1 sheet of notes in your own writing. Please show all your work and leave your answers in exact form (for example, $10 \ln 2$, $\sqrt{79}$, and $\frac{5}{6}\pi$ are in exact form).

NOTE: All work that you hand in must be on the exam sheets. Use the reverse sides if you need extra space. Please box your final answers. Cross out any work that you don't want considered. No partial credit will be given for any work that is incorrect, illegible, or unreadable.

1. (15 points) First simplify (this is required for credit) and then find the derivative:

$$\left(\frac{x^x \sin(x)}{2^{-x}} \right)^3.$$

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2. (15 points) In this problem use the decimal approximation 0.7854 for $\pi/4$. Using the tangent line approximation, find $\tan(0.8)$. Please be sure to show your work clearly.

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3. (20 points) Recall that the difference quotient is the slope of the chord joining $(x, f(x))$ to $(x + h, f(x + h))$. Using the definition of the derivative as the limit of the difference quotient, find the value of the limit

$$\lim_{h \rightarrow 0} \frac{\ln(\text{Arctan}(\sqrt{3} + h)) - \ln(\pi/3)}{h}.$$

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4. (25 points) You have a ladder of unknown length ℓ . You are pulling it away from a wall so that one end of the ladder is dragging along the ground at 0.4 m/sec. At the moment when that end of the ladder is 9 m away from the wall, the other end of the ladder is moving down the wall at 0.3 m/sec. Find ℓ . (Assume that the wall is vertical, the ground is level, and the lower end of the ladder is being pulled directly away from the wall at a right angle to the wall.)

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5. (25 points) Just as a circle consists of all points at a constant distance from the center, similarly an ellipse is defined as all points such that the sum of the distances from two foci is constant. Suppose that the ellipse E has foci at the origin and at $(5, 10)$ and passes through the point $(8, 6)$.

(a) Write an equation satisfied by the points (x, y) on E .

(b) Find the slope of the tangent line to E at the point $(8, 6)$.

(c) If you move along E so that the x -coordinate increases by $1/100$, that is, to 8.01 , by how much does the y -coordinate increase? Use the tangent line approximation.