Math 124 (Pezzoli)
Fall 2015
Midterm #2 (80 points)

Name

TA:

Section:

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Instructions:

- Your exam should contain 4 problems and 6 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Your work needs to be neat and legible.
- You **MUST** show work for credit. No credit for answers only.
- Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $\pi, \sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $3.1415, 1.4142, 0.6931$ are NOT in exact form.
- You are allowed one 8.5×11 sheet of notes (both sides). Make sure your calculator is in radian mode.

Problem #1 (15 pts)

Problem #2 (15 pts)

Problem #3 (15 pts)

Problem #4 (15 pts)

TOTAL (60 pts)

Problem 1 Compute the derivatives of the following functions. You do not need to simplify.

a) (5 points) $f(x) = \left(\frac{x}{2}\right)^{x^3+x-1}$

$y = \left(\frac{x}{2}\right)^{x^3+x-1}$, $\ln(y) = (x^3+x-1)\cdot\ln\left(\frac{x}{2}\right)$, $\frac{y'}{y} = (3x^2+1)\ln\left(\frac{x}{2}\right) + (x^3+x-1)\cdot\frac{1}{\frac{x}{2}}\cdot\frac{1}{2}$

$= (3x^2+1)\ln\left(\frac{x}{2}\right) + \frac{x^3+x-1}{x}$, so $y' = \left(\frac{x}{2}\right)^{x^3+x-1}\cdot\left[(3x^2+1)\ln\left(\frac{x}{2}\right) + \frac{x^3+x-1}{x}\right]$

or write $\left(\frac{x}{2}\right)^{x^3+x-1} = e^{(x^3+x-1)\cdot\ln\left(\frac{x}{2}\right)}$ and use chain rule

b) (5 points) $h(x) = \ln\left(\sqrt{\frac{3x+2}{x-1}}\right)$.

$h'(x) = \frac{1}{\sqrt{\frac{3x+2}{x-1}}}\cdot\frac{1}{2\sqrt{\frac{3x+2}{x-1}}}\cdot\frac{3(x-1)-(3x+2)}{(x-1)^2} = \frac{x-1}{2(3x+2)}\,\frac{-5}{(x-1)^2} = -\frac{5}{2(3x+2)(x-1)}$

c) (5 points) $y(x)$ defined by the implicit equation $xy^2+yx^2 = 2$ (your answer should be an expression in x and y) .

$y^2 + 2xyy' + y'x^2 + 2yx = 0$, $y'(2xy+x^2) = -y^2 - 2yx$

$y' = \frac{-y^2-2yx}{2xy+x^2}$

Problem 2 (15 points) Use linear approximation to estimate $(8.2)^{\frac{2}{3}}$. Give an exact answer.

2 pt   2 pt   2 pt

$f(x) = x^{2/3}$ $a = 8$ $f'(x) = \frac{2}{3} x^{-1/3}$

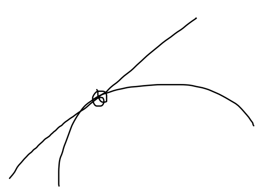
$L_f = 8^{2/3} + \frac{2}{3} 8^{-1/3} (x-8) = 4 + \frac{1}{3}(x-8)$ 2 pt

$L_f(8.2) = 4 + \frac{1}{3}(8.2-8) = 4 + \frac{0.2}{3} = 4 + \frac{2}{30} = \frac{122}{30}$

$= \frac{61}{15}$ 2 pt

Do you think the estimate you have calculated above is an underestimate or an overestimate ? Justify your answer.

$f''(x) = -\frac{2}{9} x^{-\frac{4}{3}}$, $f''(8) < 0$ so f is concave up around 8 so I think it is an overestimate



Problem 4 (15 points) A curve C has parametric equations:

$$x = t^2 + 3t$$

$$y = t^2 + 2t + 2$$

for $-\infty < t < \infty$.

(a) Find the equations of all the tangents to C that pass through the point $Q = (0, 1)$.

$Q\,(x(t)\ y(t))$ = point of tangency

m = slope of tangent line

$$m = \frac{y(t) - 1}{x(t)} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{t^2 + 2t + 2 - 1}{t^2 + 3t} = \frac{2t + 2}{2t + 3}, \quad (t^2 + 2t + 1)(2t + 3) = (2t + 2)(t^2 + 3t)$$

$$2t^3 + 4t^2 + 2t + 3t^2 + 6t + 3 = 2t^3 + 2t^2 + 6t^2 + 6t$$

$t^2 - 2t - 3 = 0$, $t = 3, -1$

for $t = 3$ $m = \frac{8}{9}$ $y = 1 + \frac{8}{9}x$ is one tangent line

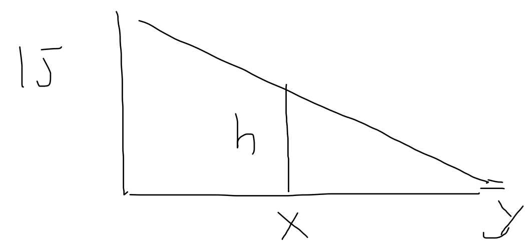
for $t = -1$ $m = 0$ $y = 1$ is another tangent line

(b) Find all points P on the curve such that the tangent line at P is horizontal.

$$\frac{dy}{dx} = \frac{2t + 2}{2t + 3}$$

We want $\frac{dy}{dx} = 0$ so $t = -1$

Problem 4 (15 points) A street light is mounted at the top of a 15-feet tall pole. An ice sculpture is being moved away from the pole with a speed of 3 feet/min along a straight path. The ice sculpture is slowly melting and its height decreases at a constant rate of 0.2 feet / min. How fast is the tip of the sculpture's shadow moving, when the sculpture is 10 feet from the pole and has a height of 5 feet ?



$x(t)=$ distance of sculpture from pole

$y(t)=$ distance of shadow tip from pole

$h(t)=$ height of sculpture

$$(*)\ \frac{y-x}{h}=\frac{y}{15},\quad 15(y-x)=y\cdot h \qquad \text{take } \frac{d}{dt}$$

$$15(y'-x')=y'h+yh' \qquad \text{plug in values:} \quad x'=3,\ h=5,\ x=10,\ h'=-0.2$$

3 pt

$$15(y'-3)=y'\cdot 5-y\cdot 0.2$$

Find y from $(*)$ $\quad \frac{y-10}{5}=\frac{y}{15},\quad y=15$

$$10y'=45-15\cdot 0.2=4.2 \text{ feet/min}$$

OR

$$\frac{y-x}{h}=\frac{y}{15},\ \text{differentiate}\ \frac{(y'-x')h-h'(y-x)}{h^2}=\frac{y'}{15}$$

$$\frac{(y'-3)5+0.2(y-10)}{25}=\frac{y'}{15},\quad y=15$$

$$5y'-\frac{5}{3}y'=15-0.2\cdot 5$$

$$y'=\frac{14*3}{10}=4.2 \text{ feet/min}$$