Name____

Math 124 **Second Midterm** **8:30 a.m., Nov. 24, 2015**

(80 minutes — 100 points — a TI-30X calculator and 1 sheet of notes are permitted)

1. (a) (10 points) Find the derivative of

$$x^{(xe^{2x})}.$$

(b) (10 points) First simplify, and then find the derivative of

$$\frac{\ln^3(\sin^2(x))}{\sqrt{\ln(\sin(x))}}.$$

(Recall that $\sin^n(x)$ means $(\sin(x))^n$ and $\ln^n(x)$ means $(\ln(x))^n$.)

(CONTINUE TO NEXT PAGE)

(c) (10 points) Recall that the difference quotient is the slope of the secant line joining $(x, f(x))$ to $(x+h, f(x+h))$. Using the definition of the derivative as the limit of a difference quotient, find the value of

$$\lim_{h \longrightarrow 0} \frac{\operatorname{Arctan}(\sqrt{36+h}) - \operatorname{Arctan}(6)}{h}.$$

(d) (15 points) Find all critical numbers in the interval $[-\frac{1}{2}, \frac{3}{2}]$ for the function $|\cos(\pi x) - \frac{1}{2}|$.

(CONTINUE TO NEXT PAGE)

2. (25 points) You are moving toward a building of height h. Let $u(t)$ be your distance from the bottom of the building at time t, and let $\theta(t)$ be the angle above the horizontal of your line of sight to the top of the building.

(a) Find a formula for u in terms of h and θ.

(b) If at any time t you knew both u and θ, then you could use the formula in part (a) to solve for h. However, you have no idea what u is. Rather, you are able to measure θ and the rate of change of θ, and you know that your speed is 30 miles/hour $=$ 44 ft/sec. Suppose that at some moment you measure that θ is equal to $60°$ and is increasing at 0.05 rad/sec. Find h. Please show your work clearly.

(CONTINUE TO NEXT PAGE)

3. (30 points) You are building a conical container of height h and base radius r. The base has area πr^2, and the material used for it costs \$10 per square foot. The lateral side has area $\pi r\sqrt{r^2+h^2}$, and the material used for it costs \$15 per square foot. If you take $r = 3$ ft and $h = 4$ ft, then the cost in dollars is $10 \cdot \pi \cdot 3^2 + 15 \cdot \pi \cdot 3\sqrt{3^2+4^2} = 315\pi$. Suppose you now want to increase h to 4.1 ft. To keep the cost the same, you'll have to decrease r. What will you have to decrease r to? Use the tangent line approximation, and please show all your work clearly.

MIDTERM ANSWERS, Nov. 24, 2015

1. (a) If y is the function given, then $\ln(y) = xe^{2x}\ln(x)$, and so $y'/y = e^{2x} + 2xe^{2x}\ln(x) + e^{2x}\ln(x)$. We get $y' = x^{(xe^{2x})}(e^{2x} + 2xe^{2x}\ln(x) + e^{2x}\ln(x))$.

(b) The function simplifies to $8\ln^{5/2}(\sin(x))$, having derivative $20(\ln(\sin(x)))^{3/2}\frac{\cos(x)}{\sin(x)}$.

(c) $\frac{d}{dx}\mathrm{Arctan}(\sqrt{x}) = \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}$; when $x = 36$ this is $\frac{1}{37}\cdot\frac{1}{2\cdot 6} = \frac{1}{444}$.

(d) The function has corners when the expression inside the absolute values is zero, and it has local maxima when that expression has zero derivative; there are two of each kind of critical value, at $\pm\frac{1}{3},\ 0,\ 1$.

2. (a) $u = h\cot(\theta)$; (b) $\frac{du}{dt} = -h\csc^2(\theta)\cdot\frac{d\theta}{dt} = -h\cdot\frac{4}{3}\cdot 0.05$, and the left hand side is -44. So $h = -3\cdot 44/(-0.2) = 660$ ft.

3. This problem is similar to #8 on HW18 and also #4 on the practice exam. Our equation is $10\pi r^2 + 15\pi r\sqrt{r^2+h^2} = 315\pi$. Dividing through by π and taking d/dh of both sides, we get $20rr' + 15r'\sqrt{r^2+h^2} + 15r(r^2+h^2)^{-1/2}(rr'+h) = 0$. We substitute $r = 3$, $h = 4$ and solve for r', getting $60r' + 15r'\cdot 5 + (45/5)(3r'+4) = 0$, or $162r' = -36$, and so $r' = -2/9$. Then the new value of r is $3 + 0.1(-2/9) = 2.9778$ ft.