# Math 124 Midterm #2 Autumn 2013 (Collingwood)

Print Your Name

Signature

Student ID Number

## !!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 4 questions and 7 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. **Unless stated otherwise**, you may calculate any derivatives using rules discussed in class thusfar.
3. **Unless stated otherwise**, you need not algebraically simplify your final answers.
4. **Unless stated otherwise**, ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
5. The entire exam is worth 50 points. Point values for problems may vary and these are clearly indicated. You have 80 minutes for this exam.
6. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
7. A scientific calculator is allowed. No calculators with graphing, symbolic or calculus features are allowed. One sheet of handwritten notes allowed, 8.5 x 11, both sides. Make sure your calculator is in radian mode.

| Problem | Total Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 17 | |
| 3 | 9 | |
| 4 | 12 | |
| Total | 50 | |

1. (12 points) Find the derivatives of the following functions. You do not have to simplify. Your final answers must give the derivative in terms of $x$. Use any rules you wish.

   (a) $y = \dfrac{x}{x^2+2}, \quad \dfrac{dy}{dx} =$

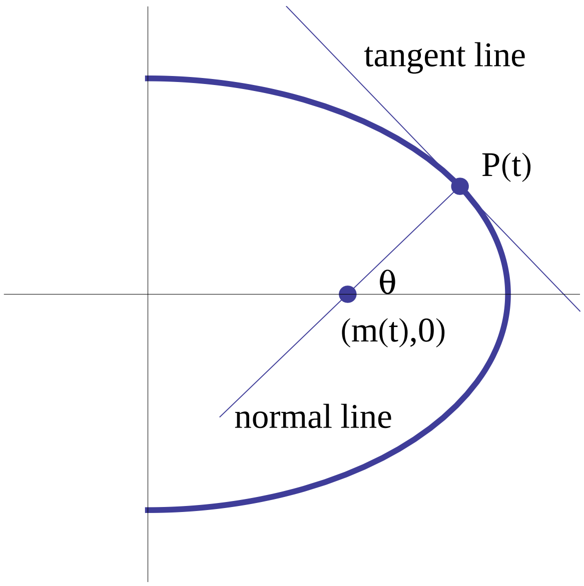
   (b) $y = e^{\sin(\sqrt{x})} + (\sin(\sqrt{x}))^e, \quad \dfrac{dy}{dx} =$

   (c) $y = \tan(x)^{\ln(x)}, \quad \dfrac{dy}{dx} =$

2. (17 points) An object is moving in the plane according to the parametric equations

$$\begin{aligned} x(t) &= 5\sin(\pi t) \\ y(t) &= 3\cos(\pi t) \end{aligned}$$

for $0 \le t \le 1$, where time units are "seconds" and the units on the coordinate axes are "feet". The path the object travels will be a portion of an ellipse, as pictured. The location of the object at time $t$ will be denoted $P(t) = (x(t), y(t))$. A laser beam projects from the object in a direction perpendicular to the tangent line along what is called a *normal line*. If $t \neq 1/2$, the normal line will cross the $x$-axis at a point $(m(t), 0)$.




(a) Write the equation of the normal line through the point $P(t)$, when $0 < t < 1$.

(b) Find a formula for $m(t)$, when $t \neq 1/2$, as a function of $t$. Use this to calculate $\lim_{t\to\frac{1}{2}} m(t)$.

2. continued.

(c) Consider the acute angle $\theta$ between the normal line and the $x$-axis (as pictured). At what rate is $\theta$ changing when $t = 1/2$?

2. continued.

(d) Find the critical numbers for the speed of the object on the time interval $[0, 1]$ and use this to find the minimum and maximum speed of the object.

3. (9 points) The length $L$ of a rectangle increases by 3 ft/min while the width $W$ decreases by 2 ft/min. When the length is 15 ft and the width is 8 ft, what is the rate at which the following are changing. (Make sure to state whether the rate is increasing or decreasing and include units.)

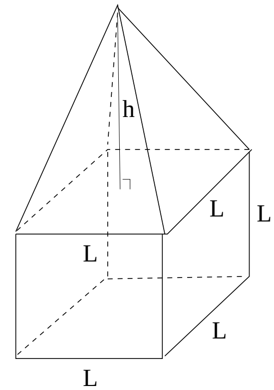
   (a) The area $A$.

   (b) The length of the diagonal $D$.

4. (12 points) A tent is constructed with a pyramid top and square sides. The surface area is given by the equation

$$S = 4L^2 + L\sqrt{4h^2 + L^2}$$

where $h$ and $L$ are as in the picture. (*You can assume this formula for the problem.*)




Notice that if $L = 3, h = 2$, the tent has surface area 51 . Any similar tent of surface area 51 would have dimensions satisfying the equation:

$$51 = 4L^2 + L\sqrt{4h^2 + L^2}$$

(a) Calculate the implicit derivative $\frac{dL}{dh}$ when $L = 3, h = 2$. Leave your answer in EXACT FORM; do not round.

(b) Suppose we want to construct another similar tent of surface area 51 with $h = 1.8$. Use linear approximation to estimate the value of $L$. Leave your answer in EXACT FORM; do not round.