1. **[4 points per part]** Compute each limit. You may use any techniques you know.

   If a limit does not exist, write DNE, ∞, or $-\infty$ as appropriate.

   (a) $\lim_{x\to 5} \underbrace{\sqrt{2^x + 7x}}_{\text{continuous at } x=5} = \sqrt{2^5 + 35} = \boxed{\sqrt{67}}$

   (b) $\lim_{x\to 2} \frac{x-3}{(x-2)^2}$

   numerator approaches -1

   denominator approaches 0 (positive from both directions)

   $\text{limit} = \boxed{-\infty}$

   (c) $\underbrace{\lim_{t\to a} \frac{\sec(t) - \sec(a)}{t-a}}$ (Your answer will include the constant a.)

   this is the definition of $f'(a)$ where $f(t) = \sec(t)$

   $\boxed{f'(a) = \sec(a)\tan(a)}$

   (d) $\lim_{x\to\infty} \sin\left(\frac{\pi x^4 + 3}{3x^4 + \pi}\right)$

   $= \sin\left(\lim_{x\to\infty} \frac{\pi x^4 + 3}{3x^4 + \pi}\right) = \sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

2. **[8 points]** Consider the curve $y = \frac{3}{x}$.

Give the equation for a tangent line to this curve which has a y-intercept of 8.

$$\frac{dy}{dx} = \frac{-3}{x^2}$$

Tangent line at $\left(a, \frac{3}{a}\right)$: $y = \frac{-3}{a^2}(x-a) + \frac{3}{a}$

$$= \frac{-3x}{a^2} + \underbrace{\frac{6}{a}}_{y\text{-intercept}}$$

$$\frac{6}{a} = 8 \rightarrow a = \frac{3}{4}$$

$$y = \frac{-3x}{\left(\frac{3}{4}\right)^2} + \frac{6}{\frac{3}{4}} = \boxed{\frac{-16}{3}x + 8}$$

3. **[8 points]** Let $f(x) = x^3 e^x + \sqrt{x}$. Compute $f''(x)$.

$$f'(x) = 3x^2 e^x + x^3 e^x + \frac{1}{2} x^{-1/2}$$

$$f''(x) = 6xe^x + 3x^2 e^x + 3x^2 e^x + x^3 e^x - \frac{1}{4} x^{-3/2}$$

$$= \boxed{\left(x^3 + 6x^2 + 6x\right)e^x - \frac{1}{4\sqrt{x^3}}}$$

4. Consider the following piecewise function:

$$f(x) = \begin{cases} \dfrac{x^2 + ax - 21}{x - 3} & \text{if } x < 3 \\ b & \text{if } x = 3 \\ 3\cos(x) + c & \text{if } x > 3 \end{cases}$$

(a) **[8 points]** Determine constants a, b, and c so that $f(x)$ is continuous at $x = 3$.

$$\lim_{x\to 3^-} \frac{x^2+ax-21 \to 3a-12}{x-3 \to 0} \quad \text{DNE unless } 3a-12=0 \to a=4$$

$$\lim_{x\to 3^-} \frac{x^2+4x-21}{x-3} = \lim_{x\to 3^-} \frac{(x+7)\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x\to 3^-}(x+7) = 10$$

$f(3) = b$

$$\lim_{x\to 3^+} 3\cos(x)+c = 3\cos(3)+c$$

Want $10 = b = 3\cos(3)+c$

So $\boxed{a=4,\ b=10,\ c=10-3\cos(3)}$

(b) **[8 points]** Find $f'(x)$. (Let a, b, and c be the values you found in part (a).)

(Note: your answer will be a piecewise function, just like $f(x)$.)

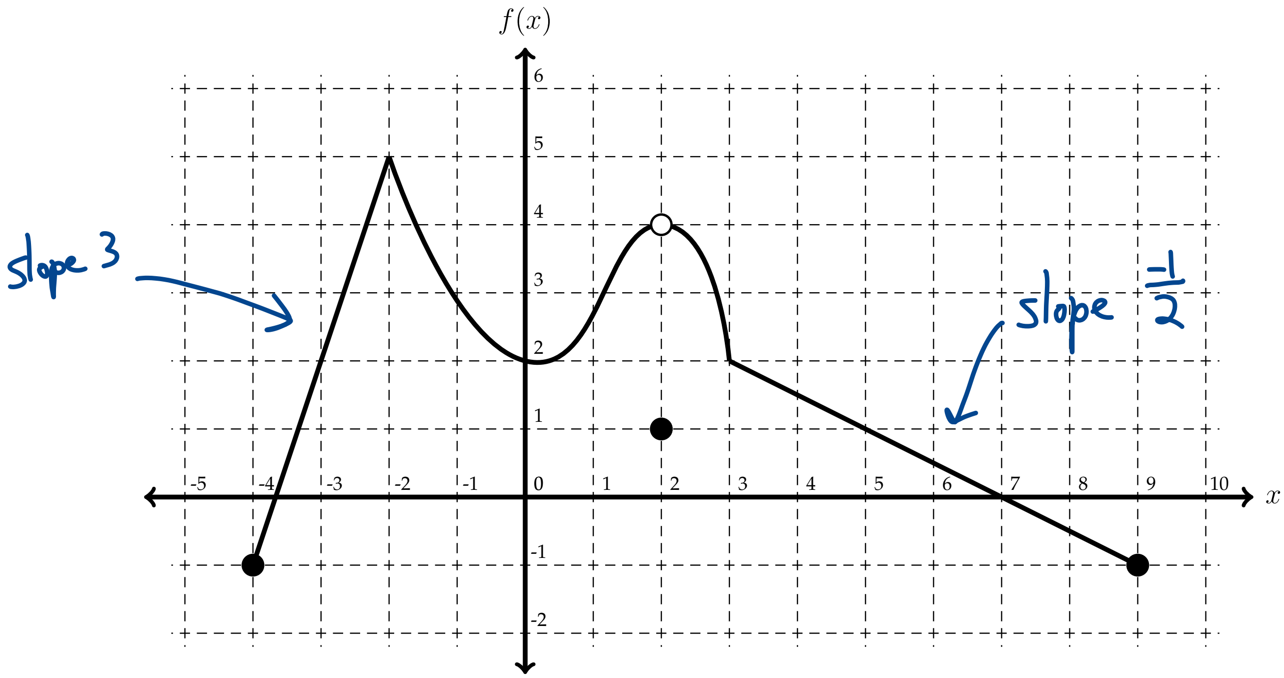
For $x<3$, $f(x)=x+7$, so $f'(x)=1$

For $x>3$, $f(x)=3\cos(x)+10-3\cos(3)$, so $f'(x)=-3\sin(x)$

f is not differentiable at $x=3$ $\left(\lim_{x\to 3^+} f'(x) = -3\sin(3), \text{ not } 1\right)$

So $$\boxed{f'(x) = \begin{cases} 1 & \text{if } x<3 \\ -3\sin(x) & \text{if } x>3 \end{cases}}$$

5. The graph of $f(x)$ is shown below.



Cool graph, right? Use it to answer the following questions.

(a) **[3 points]** Compute $\lim_{x\to 2}[\underbrace{f(x)}_{4} \cdot \underbrace{f(x+1)}_{2}]$. $= \boxed{8}$

(b) **[3 points]** List all values in the open interval $(-4, 9)$ where f is *not* differentiable.

$x = \boxed{-2,\ 2,\ 3}$

cusp discontinuity cusp

(c) **[3 points]** Compute $\lim_{h\to 0^+} \underbrace{\frac{f(3+h)-2}{h}}_{f'(3) \text{ from the right}}$. $= \boxed{\frac{-1}{2}}$

f(3)

(d) **[3 points]** Let $g(x) = xf(x)$. What is $g'(-3)$?

$g'(x) = f(x) + xf'(x)$

$g'(-3) = \underbrace{f(-3)}_{2} - 3\underbrace{f'(-3)}_{3} = \boxed{-7}$