**Math 124 Section BD** **Midterm I, February 1st** **Winter 2022**

HONOR STATEMENT
I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

| Name | Signature |
|---|---|
| | |

Student ID #

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | $\sum$ |
|---|---|---|---|---|---|---|---|---|
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 80 |
| | | | | | | | | |

- You have 80 minutes for 8 problems. Check your copy of the exam for completeness.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- Calculators may only have basic functions, but no graphing or differentiation functions.
- Justify all your answers and show your work for credit.
- Some credit is given for adhering to formal aspects such as keeping the limit symbol until you take the limit, setting correct parentheses etc.
- All answers must be exact, no rounding.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

**Problem 1.** Find the limit of the following expression. Your answer must be a real number (exact value), $\infty$, $-\infty$, or DNE, whatever fits **best**. Justify all your work.

$$\lim_{x \to 6} \frac{x^2 - 4x - 12}{x^2 - 5x - 6}.$$

**Problem 2.** Find the limit of the following expression. Your answer must be a real number (exact value), $\infty$, $-\infty$, or DNE, whatever fits best. Justify all your work.

$$\lim_{x \to 3} \arctan\left(\frac{1}{(x-3)^2}\right).$$

**Problem 3.** The function

$$f(x) = \begin{cases} ax^2 - a + 10 & , \quad x < 1 \\ \\ 12 - 2\sqrt{x} & , \quad x \geq 1 \end{cases}$$

is continuous at $x = 1$ (you do not need to show this). Determine the constant $a$ so that $f$ is differentiable at all places in $\mathbb{R}$.

**Problem 4.** Use the definition of a derivative (the limit-version, no differentiation rules) to find the derivative function of $f(x) = \frac{1}{\sqrt{x}}$ at $x = 3$. Show all your work and give exact values.

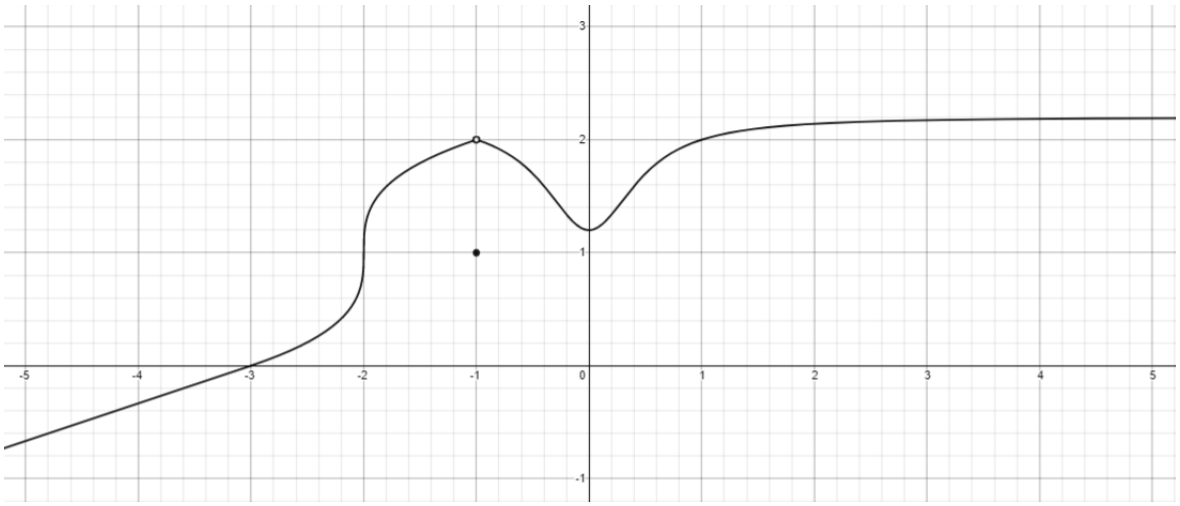
**Problem 5.** Find the points on the curve

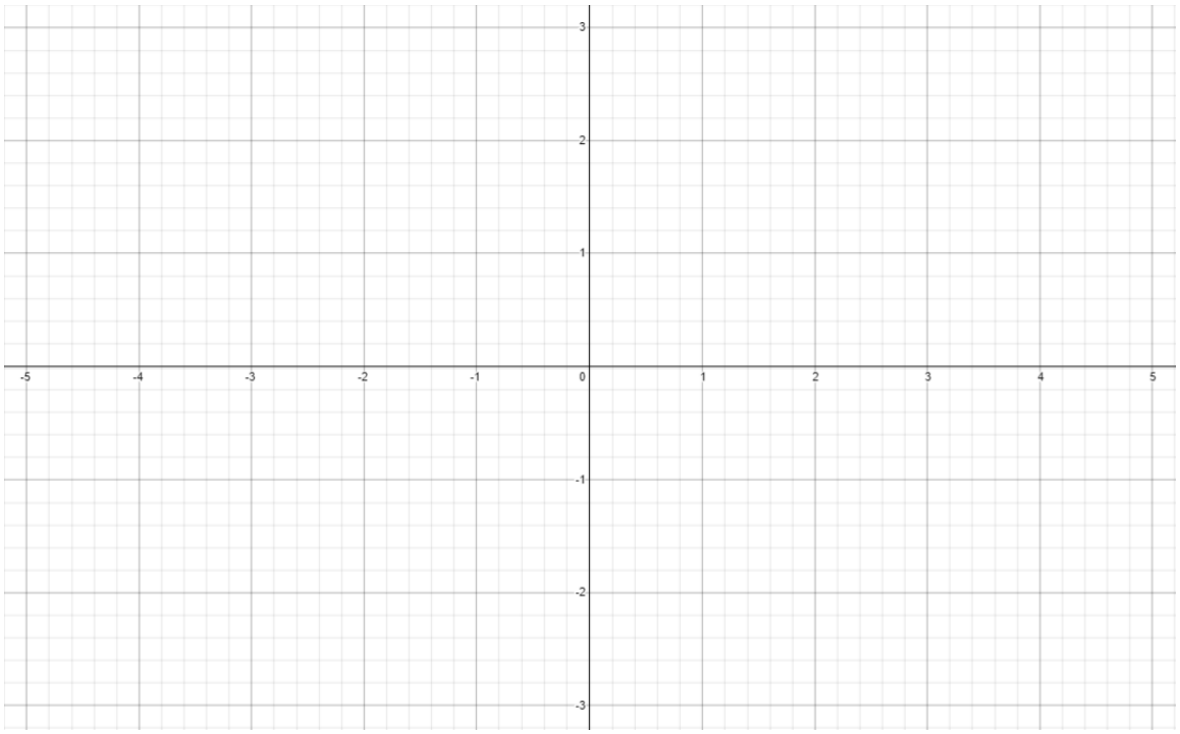
$$y = 2x^3 + 3x^2 - 12x + 1$$

where the tangent line is horizontal.

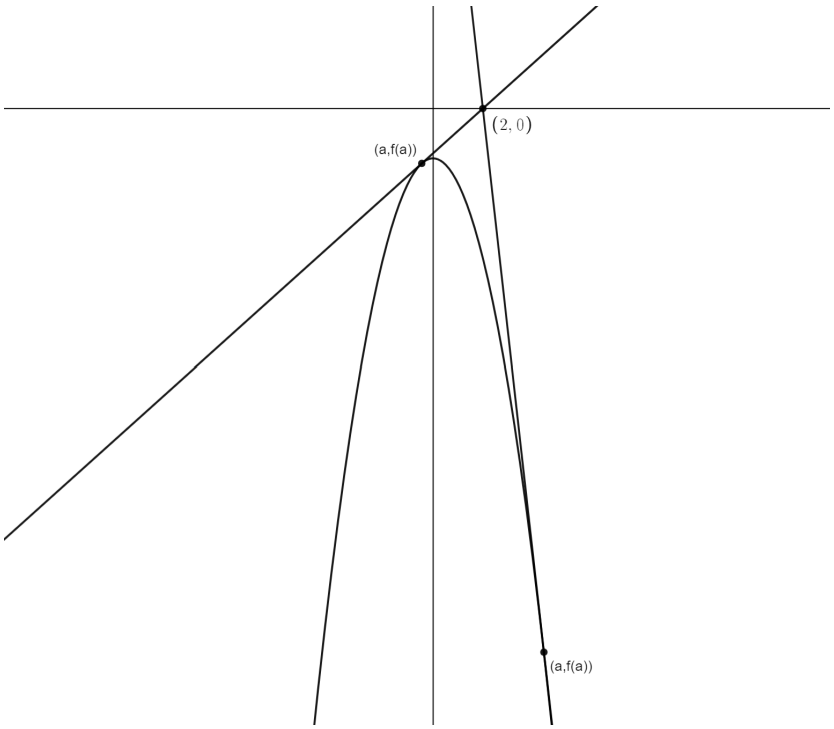
**Problem 6.** Consider the following graph of the function $f(x)$. In the given blank coordinate system sketch the graph of $f'(x)$. Be sure to correctly sketch

- Where $f'(x)$ is positive/negative or equal to 0.
- Where $f'(x)$ is constant and what value that constant is.
- Where $f'(x)$ is increasing and where it is decreasing.
- Where $f'(x)$ is not defined.



**Problem 7.** Consider the function $f(x) = -x^2 - 2$. Find the values both values for $a$ so that the tangent lines through $(a, f(a))$ to the graph of $f$ pass through the point $(2, 0)$ as shown in the sketch below.



**Problem 8.** The population of bacteria in a petri dish can be modeled by

$$N(t) = \frac{3250t + 1}{t + 1},$$

where $t$ measures the time in days ($t \geq 0$). Show these **TWO** features of the population:

(a) The population of the bacteria is always (for all $t \geq 0$) growing.

(b) But the number of bacteria is nearing a certain number when we observe the population for a very long time.