124_fall22_midtermI

Saturday, October 22, 2022 7:06 PM

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Math 124 Section **Midterm I, October 25th** **Autumn 2022**

HONOR STATEMENT

I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

KEY

1.	2.	3.	4.	5.	6.	7.	8.	$\sum$
10	10	10	10	10	10	10	10	80

- You have 80 minutes for 8 problems. Check your copy of the exam for completeness.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- Calculator : TI 30 X.
- Justify all your answers and show your work for credit.
- Some credit is given for adhering to formal aspects such as keeping the limit symbol until you take the limit, setting correct parentheses etc.
- All answers must be exact, no rounding.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

Problem 1. Find the limit of the following expression. Your answer must be a real number (exact value), ∞, $-\infty$, or DNE, whatever fits best. Justify all your work algebraically.

$$\lim_{x\to 2^+}\left(\frac{\sqrt{x^2-4}}{x-2}\right).$$

$$\lim_{x\to 2^+}\frac{\sqrt{x^2-4}}{x-2} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 2^+}\sqrt{\frac{\cancel{(x-2)}(x+2)}{(x-2)^{\cancel{2}}}} = \lim_{x\to 2^+}\sqrt{\frac{x+2}{x-2}} \overset{\left[\frac{c^+}{0^+}\right]}{=} \boxed{\infty}$$

Problem 2. Find the limit of the following expression. Your answer must be a real number (exact value), ∞, $-\infty$, or DNE, whatever fits **best**. Justify all your work algebraically.

$$\lim_{x\to 0}\left(\frac{1}{x\sqrt{x+1}}-\frac{1}{x}\right).$$

$$\lim_{x\to 0}\left(\frac{1}{x\sqrt{x+1}}-\frac{1}{x}\right) \overset{[\infty-\infty]}{=} \lim_{x\to 0}\frac{1-\sqrt{x+1}}{x\sqrt{x+1}} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0}\frac{(1-\sqrt{x+1})(1+\sqrt{x+1})}{x\sqrt{x+1}\,(1+\sqrt{x+1})}$$

$$= \lim_{x\to 0}\frac{1-x-1}{x\sqrt{x+1}\,(1+\sqrt{x+1})} = \lim_{x\to 0}\frac{-\cancel{x}^{\,1}}{\cancel{x}\sqrt{x+1}\,(1+\sqrt{x+1})}$$

$$= \lim_{x\to 0}\frac{-1}{\sqrt{x+1}\,(1+\sqrt{x+1})} \underset{DSP}{\overset{\left[\frac{c}{c}\right]}{=}} \frac{-1}{1\cdot 2} \boxed{= -\frac{1}{2}}$$

Problem 3. Find the derivative of $f(x) = \cos(x)\sin(x) + 3e^x$

$$\boxed{f'(x) = -\sin x \cdot \sin x + \cos x \cdot \cos x + 3e^x}$$

Problem 4. Find the tangent line equation to the graph of

$$f(x) = \frac{x+1}{\sqrt{x}+2} \text{ at } x = 1.$$

Do not round.

point of tangency : $\left(1, \frac{2}{3}\right)$

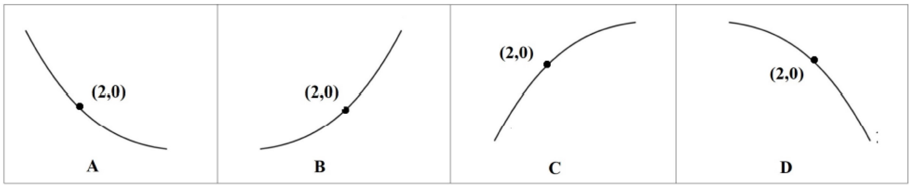
slope : $f'(x) = \dfrac{\sqrt{x}+2 - (x+1)\frac{1}{2\sqrt{x}}}{(\sqrt{x}+2)^2}$

$f'(1) = \dfrac{3-1}{3^2} = \dfrac{2}{9}$

tangent line equation :

$$\boxed{y = \frac{2}{9}(x-1) + \frac{2}{3}}$$

Problem 5.
Assume that f is a function so that $f(2) = 0$, $f'(2) = -1$, and $f''(2) = 2$. For **each** of the following options argue why it can **or** why it can not be the graph of f locally around $x = 2$.

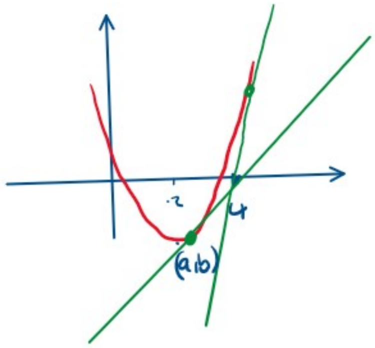


$f'(2) = -1 \Rightarrow$ f decreasing, so B, C not possible

$f''(2) = 2 \Rightarrow$ f concave up, so D not possible

$\Rightarrow$ A shows decreasing & concave up shape.

Problem 6. Consider the function $f(x) = x^2 - 4x - 1$. There are two tangent lines to the graph of $f(x)$ that have x-intercept 4. Find both points of tangency.



- pt of tangency not given
- $f'(x) = 2x - 4$

1) (a,b) on curve: $b = a^2 - 4a + 1$

2) tangent line slope: $\frac{\text{rise}}{\text{run}} = f'(a)$, so

$$\frac{b-0}{a-4} = 2a - 4 \iff b = (2a-4)(a-4)$$

3) Combine 1,2 : $a^2 - 4a + 1 = 2a^2 - 8a - 4a + 16$

$$\iff a^2 - 8a + 15 = 0$$

$$\iff (a-4)^2 = 1$$

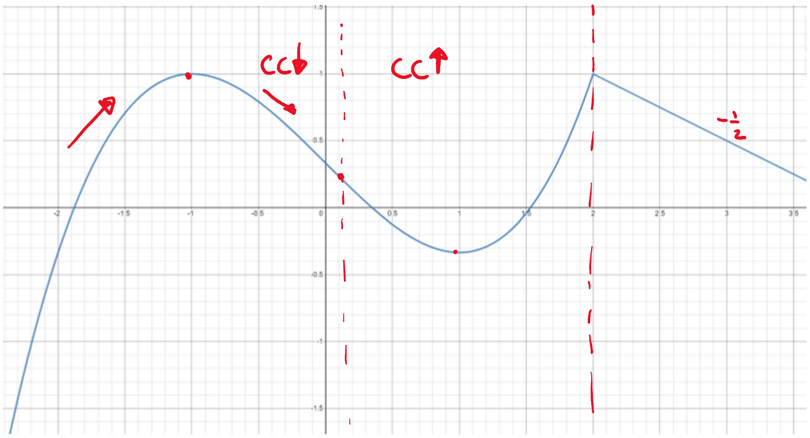
$$\iff a = 5 \text{ OR } = 3$$

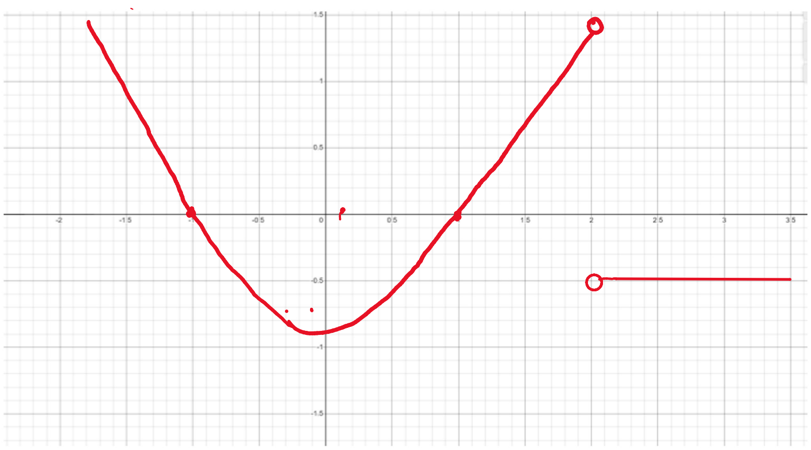
$\Rightarrow$ $a = 5$, then $b = 25 - 20 + 1 = 6$ $\rightarrow$ $\boxed{(5,6)}$

$a = 3$, then $b = 9 - 12 + 1 = -2$ $\rightarrow$ $\boxed{(3,-2)}$

Problem 7. Consider the following graph of the function $f(x)$. In the given blank coordinate system sketch the graph of $f'(x)$. Be sure to correctly sketch

- Where $f'(x)$ is positive/negative or equal to 0.
- Where $f'(x)$ is constant and what value that constant is.
- Where $f'(x)$ is increasing and where it is decreasing.
- Where $f'(x)$ is not defined.





Problem 8. The temperature of a probe in a laboratory is described by the function

$$T(x) = \frac{2x^2+1}{x^2+3},$$

where T is the temperature in Celsius and x is time in minutes. We only consider times $x \geq 0$.

(a) Why is it true that the temperature always increases?

(b) Which temperature is the probe getting closer and closer to after sitting in the laboratory for a very long time?

a) A function is increasing if its derivative is positive

$$T'(x) = \frac{4x(x^2+3)-(2x^2+1)2x}{(x^2+3)^2} = \frac{4x^3+12x-4x^3-2x}{(x^2+3)^2}$$

$$= \frac{10x}{(x^2+3)^2}.$$

For all positive times, this derivative is positive, so $T(x)$ is increasing.

b) Long time = '$x \to \infty$' :

$$\lim_{x\to\infty} \frac{2x^2+1}{x^2+3} = \lim_{x\to\infty} \frac{2+\cancel{\frac{1}{x^2}}\to 0}{1+\cancel{\frac{3}{x^2}}\to 0} = 2$$

$\Rightarrow$ The probe nears the temperature 2°C when it is observed for a very long time.