1. Your exam contains 5 questions and 6 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.

2. Unless stated otherwise, you may calculate any derivatives using rules discussed in class thusfar.

3. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.

4. Unless stated otherwise, make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.

5. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.

6. Graphing calculators are NOT allowed; scientific calculators are allowed. One sheet of handwritten notes allowed, 8.5 x 11. Make sure your calculator is in radian mode. NO SCRATCH PAPER allowed.

7. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $3\pi$, $\sqrt{2}$, $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, 0.693147 are NOT in exact form.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Total Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. (20 points; 5pts each) Determine if the following limits exist. If a limit exists, you MUST show your work or give a reason for your answer by referring to some rule or result. If the limit does not exist, you MUST explain why. Use of a calculator to “plug in values” is NOT considered justification. You will not receive full credit for an unjustified answer. Give your answers in EXACT FORM; decimal approximations will not receive full credit.

(a) \[ \lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+3)}{x-2} = \lim_{x \to 2} x + 3 = 5 \]

(b) \[ \lim_{t \to 0} \left( \frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) = \lim_{t \to 0} \frac{t - \sqrt{1+t}}{t^2 \sqrt{1+t}} = \lim_{t \to 0} \frac{1 - \sqrt{1+t}}{t \sqrt{1+t} (1 + \sqrt{1+t})} = \lim_{t \to 0} \frac{-1}{t(1+1)} = -\frac{1}{2} \]

(c) \[ \lim_{r \to 0} \left( \frac{3r^2}{3 - \sqrt{9 - r^2}} \right) = \lim_{r \to 0} \frac{3r^2 (3 + \sqrt{9 - r^2})}{(3 - \sqrt{9 - r^2})(3 + \sqrt{9 - r^2})} = \lim_{r \to 0} \frac{3r^2 (3 + \sqrt{9 - r^2})}{9 - (9 - r^2)} = \lim_{r \to 0} \frac{3r^2 (3 + \sqrt{9 - r^2})}{9 - (9 - r^2)} = 3(1+1) = 6 \]

(d) \[ \lim_{x \to 0} \left( \frac{e^x \sin(3x) + 5 \sin(3x)}{x} \right) = \lim_{x \to 0} \left( e^x + 5 \right) \left( \frac{\sin 3x}{3x} \right) \]

\[ = \lim_{x \to 0} 3(e^x + 5)^2 \left( \frac{\sin 3x}{3x} \right) = \frac{3}{3} \]

\[ = \lim_{x \to 0} 3(e^x + 5) \cdot \lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1+5) \cdot 1 = 18 \]
2. (20 points) Consider this multipart function, where \( c \) is a constant:

\[
 f(x) = \begin{cases} 
 -x + c & \text{if } x \leq 1 \\
 6 - 2x^2 & \text{if } x > 1 
\end{cases}
\]

(a) (6pts) Find a value of \( c \) so that \( f(x) \) is continuous at \( x = 1 \). Explain your reasoning.

- \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-x + c) \)
- \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (6 - 2x^2) \)

(b) (8pts) For the value of \( c \) in (a),

\[
\lim_{h \to 0^+} \frac{f(1 + h) - f(1)}{h} = \lim_{h \to 0^+} \frac{6 - 2(1+h)^2 - 4}{h} = \lim_{h \to 0^+} \frac{6 - 2 - 4h - 2h^2 - 4}{h} = \lim_{h \to 0^+} \frac{-4 - 4h}{h} = \lim_{h \to 0^+} \frac{-1 - h + 5 - y}{h}
\]

(c) (2pts) For the value of \( c \) in (a), where is the function \( f(x) \) differentiable? Explain.

\( \text{Differentiable for all } x \neq 1 \). Not diff. @ \( x = 1 \)

(d) (4pts) For the value of \( c \) in (a), sketch the graph of \( y = f'(x) \) below.

[Diagram of the graph of the function]
3. (20 points) Consider the curve \( y = f(x) = x^2 - 6x + 1 \), defined for all values of \( x \).

(a) (10pts) Find the equation of the tangent line to the curve at the point (6,1).

\[
\begin{align*}
4pt+ & ( TL: y = f'(6)(x-6) + 1 \\
4pt+ & f'(x) = 2x - 6 \\
2pt+ & f'(6) = 12 - 6 = 6 \\
2pt+ & ( y = 6(x-6) + 1 \\
\text{final} & 
\end{align*}
\]

(b) (10pts) Find the coordinates of a point \( P \) on the curve where the tangent line has \( x \)-intercept 8.

\[
\begin{align*}
& \{ TL @ (a, f(a)) \} is \\
& \quad y = f'(a)(x-a) + f(a) \\
& y = (2a-6)(x-a) + (a^2 - 6a + 1) \\
& \quad 3pt 1pt 1pt 1pt \\
& \{ TL thru (8,0): \} \\
& \quad 3pt \\
& \quad 0 = (2a-6)(8-a) + a^2 - 6a + 1 \\
& \quad 0 = 16a - 48 - 2a^2 + 6a + a^2 - 6a + 1 \\
& \quad 0 = -a^2 + 16a - 47 \\
& \quad 2pt \\
& \text{By q. Formula: } a = \frac{-16 \pm \sqrt{16^2 - 4 \cdot 47}}{-2} = \frac{-16 \pm \sqrt{256}}{-2} \\
& \quad = \frac{-16 \pm 2\sqrt{17}}{-2} = 8 \pm \sqrt{17} \\
& \text{2pts as ln} \\
& (2pts) ( Two correct answers; either ok } \\
& \text{one of the pts} \\
& (8+\sqrt{17}, f(8+\sqrt{17})) \text{ or } (8-\sqrt{17}, f(8-\sqrt{17}))
\end{align*}
\]
4. (20 points; 4pts each) The picture below labels 24 graphs using letters (a) to (x). For each of the following graphs of $f(x)$, give the letter of the graph that looks most like it could be the graph of the derivative function $f'(x)$:

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td>0</td>
</tr>
<tr>
<td>s</td>
<td>f</td>
</tr>
<tr>
<td>t</td>
<td>d</td>
</tr>
<tr>
<td>w</td>
<td>i</td>
</tr>
<tr>
<td>u</td>
<td>m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(i)</th>
<th>(j)</th>
<th>(k)</th>
<th>(l)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(m)</th>
<th>(n)</th>
<th>(o)</th>
<th>(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(q)</th>
<th>(r)</th>
<th>(s)</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(u)</th>
<th>(v)</th>
<th>(w)</th>
<th>(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. (20 points) The location of an object moving along the number line at time $t$ seconds is given by

$$d(t) = \frac{100}{5 + 4 \sin(t)} \text{ feet}$$

On this problem, give exact answers.

(a) (4pts) Find the average velocity of the object on the time interval $\left[\frac{\pi}{2}, \pi\right]$.

$$\frac{d(\pi) - d\left(\frac{\pi}{2}\right)}{\pi - \frac{\pi}{2}} = \frac{100}{5 + 4 \cdot 0} - \frac{100}{5 + 4 \cdot 1} = \frac{100}{5} - \frac{100}{9} = \frac{5}{2}$$

(b) (6pts) Find the instantaneous velocity at time $t = \pi$ seconds.

$$d'(\pi) = \frac{-100(4\cos t)}{(5 + 4\sin t)^2}$$

$$= \frac{-100(4\cdot(-1))}{(5 + 4\cdot0)^2} = \frac{400}{25} = 16 \text{ ft/s}$$

(c) (2pts) At time $t = \pi$, is the object moving left or right? Explain.

Right since $d'(\pi) > 0$

(d) (8pts) On the time domain $[0, 6]$, find all of the times when the object is moving left and all of the times when the object is moving right.

$$d'(t) = 0 \Rightarrow \frac{-100 \cos t}{(5 + 4 \sin t)^2} = \cos t = 0$$

$$t = \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \text{ or } \frac{\pi}{2} + k\pi \text{ or } \frac{3\pi}{2} + k\pi$$

$$d'(\frac{\pi}{2}) < 0 \Rightarrow \text{ left}$$

$$d'(\frac{3\pi}{2}) > 0 \Rightarrow \text{ right}$$

$$d'(2\pi) = 0$$

$$d'(0) > 0$$

$$d'(1) > 0$$

$$d'(\frac{\pi}{2}) = 0$$

$$d'(1) < 0$$

$$d'(2) = 0$$

$$d'(\frac{3\pi}{2}) = 0$$

$$d'(2) < 0$$

$$d'(5) = 0$$

$$d'(6) > 0$$