MATH 124 Midterm 1

October 25, 2022

Name: **Solutions**

Student #: _____

Problem:	1	2	3	4	5	Total
Points:	15	15	15	8	7	60

INSTRUCTIONS:

- You have 80 minutes to take the test.

- There are 5 problems. Make sure you have all of them.

- Write your solution below the problem. There is scratch paper at the back of the test.

- The test is double-sided. Make sure you are reading the backs of pages!

- Unless otherwise stated, **show all your work for full credit**.

- Unless otherwise stated, all answers should be exact, without rounding.

- You are allowed to use one 8.5"×11" sheet of notes, front and back.

- You can use a TI-30X IIS calculator. No other calculator is allowed.

TIPS:

- The number of points a question is worth is not correlated to its difficulty.

- Don't spend too much time on one problem if you haven't looked at the rest of the test.

- There is partial credit. Even if you can't fully solve a problem, explaining your progress might get you a significant number of points.

- Make sure your calculator is in radians!!!

Good luck!

1. For each of these questions, find the limit. If the limit does not exist, state whether the limit is ∞ or $-\infty$ if either applies; if neither applies write "DNE".

   (a) (5 points) $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

   $$\lim_{x \to 1} \frac{(x+2)(x-1)}{(x+1)(x-1)}$$

   $$\boxed{\frac{3}{2}}$$

   (b) (5 points) $\lim\limits_{x \to \infty} \dfrac{3x^2 + x + 1}{7x^2 - x - 8}$

   $$\lim_{x \to \infty} \frac{3x^2 + x + 1}{7x^2 - x - 8} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

   $$\lim_{x \to \infty} \frac{3 + \frac{1}{x} + \frac{1}{x^2}}{7 - \frac{1}{x} - \frac{8}{x^2}}$$

   $$\boxed{\frac{3}{7}}$$

   (c) (5 points) $\lim\limits_{x \to 0^+} \dfrac{\cos(\pi - x)}{x}$

   numerator $\rightarrow \cos \pi = -1$

   denominator $\rightarrow 0^+$

   $$\boxed{-\infty}$$

2. (a) (5 points) Let $f(x) = \sin x + \cos x - \sqrt{x} - \dfrac{1}{x}$. Find $f'(x)$.

$$f'(x) = \cos x - \sin x - \frac{1}{2\sqrt{x}} + \frac{1}{x^2}$$

(b) (5 points) Let $f(x) = (4x^2 + x)e^x$. Find $f'(x)$.

$$f'(x) = \left(\frac{d}{dx}(4x^2+x)\right)e^x + (4x^2+x)\frac{d}{dx}e^x$$
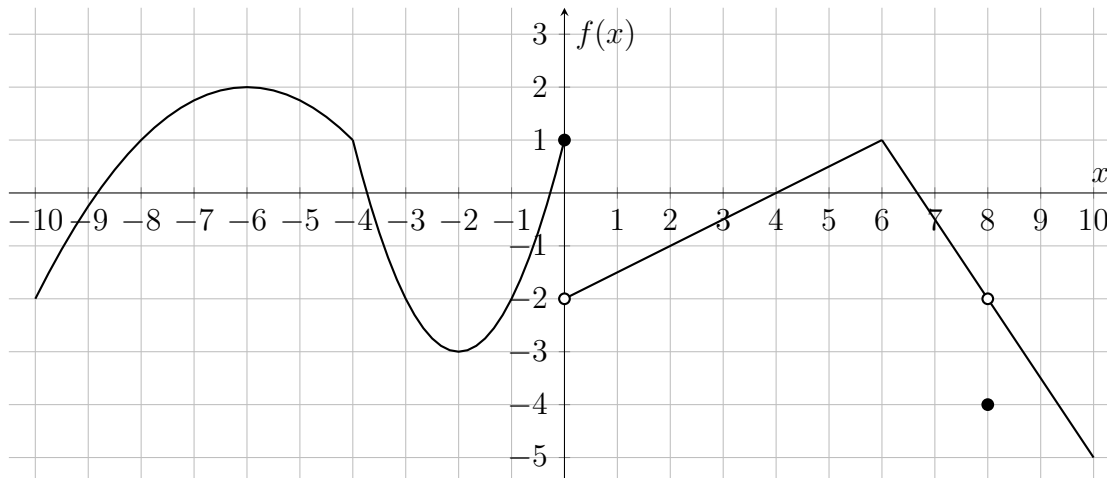
$$= (8x+1)e^x + (4x^2+x)e^x$$

(c) (5 points) Find the equation of the tangent line to the graph of $y = x^3 - 3x - 1$ at the point $(2, 1)$.

$$y' = 3x^2 - 3$$

$$y'(2) = 9$$

$$y - 1 = 9(x-2)$$

3. For this problem, **you do not have to show your work**.

Answer the following questions based on the graph of $f(x)$ shown below. If a limit does not exist, state whether that limit is ∞ or $-\infty$ if either applies; if neither applies write "DNE".

(a) (1 point) $\displaystyle\lim_{x \to 0^-} f(x)$

1

(b) (1 point) $\displaystyle\lim_{x \to 0^+} f(x)$

-2

(c) (1 point) $\displaystyle\lim_{x \to 0} f(x)$

DNE

(d) (2 points) $\displaystyle\lim_{x \to 8} f(x)$

-2

(e) (2 points) List all values of a where $f'(a) = 0$.

$-6, -2$

(f) (2 points) List all values of a where $f'(a)$ is undefined.

$-4, 0, 6, 8$

(g) (2 points) $\displaystyle\lim_{h \to 0} \frac{f(2+h)+1}{h}$

$\dfrac{1}{2}$

(h) (2 points) $\displaystyle\lim_{x \to 8} f'(x)$

$-\dfrac{3}{2}$

(i) (2 points) $\displaystyle\lim_{h \to 0^+} \frac{f(4+h)-2}{h}$ (Be careful!)

numerator $\to$ -2
denominator $\to$ 0^+

$\boxed{-\infty}$

4. Let a be a number, and let

$$f(x) = \begin{cases} 3ax - 3 & \text{if } x < 1 \\ ax^2 + x & \text{if } x \geq 1 \end{cases}$$

(a) (4 points) Find all values of a which make $f(x)$ continuous at $x = 1$. If there are none, explain why.

$$\lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$$

$$3a - 3 = a + 1 = a + 1$$

$$2a = 4$$

$$\boxed{a = 2}$$

(b) (4 points) Find all values of a which make $f(x)$ differentiable at $x = 1$. If there are none, explain why.

Since f must also be continuous at $x = 1$
$a = 2$ is the only possible candidate.

However,

$$\frac{d}{dx}(6x - 3)\Big|_{x=1} = 6$$
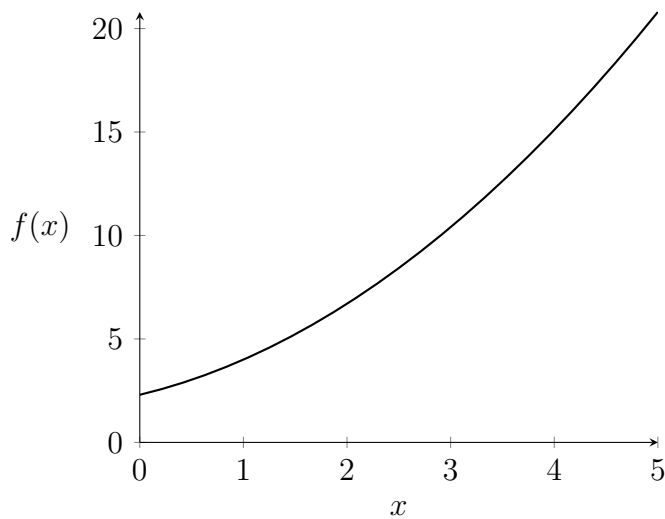
$$\frac{d}{dx}(2x^2 + x)\Big|_{x=1} = 4x + 1\Big|_{x=1} = 5$$

So $a = 2$ doesn't work.

$\boxed{\text{None}}$

5. A scientist is studying a function $f(x)$. Through experiment, they have obtained a table of approximate values of $f(x)$ and an approximate graph of $f(x)$, shown below.

x	$f(x)$
0.0	2.3
0.5	3.0
1.0	4.0
1.5	5.2
2.0	6.7
2.5	8.4
3.0	10.4
3.5	12.6
4.0	15.1
4.5	17.8
5.0	20.8

Using a computer, the scientist also approximates the function $f'(x)$. The results are shown below.

x	$f'(x)$
0.0	1.2
0.5	1.7
1.0	2.2
1.5	2.7
2.0	3.2
2.5	3.7
3.0	4.2
3.5	4.7
4.0	5.2
4.5	5.7
5.0	6.2
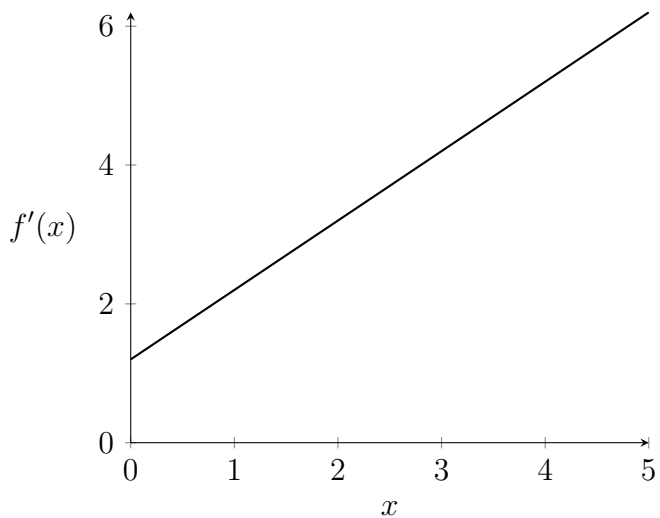
(Problem 5 continued on next page.)

(a) (3 points) The scientist believes $f(x)$ is either a quadratic function (that is, of the form $f(x) = ax^2 + bx + c$ where a, b, and c are constants) or an exponential function (that is, of the form $f(x) = ab^x$ where a and b are constants). Based on the given information, which do you think is correct? Explain your answer.

Hint: Think about what the derivative of a quadratic function looks like, and what the derivative of an exponential function looks like.

$$\frac{d}{dx}(ax^2 + bx + c) = 2ax + b, \text{ which is a line}$$

$$\frac{d}{dx}(ab^x) = a(\ln b)b^x, \text{ which is exponential}$$

The graph of f' is a line, so f is a $\boxed{\text{quadratic function}}$

(b) (4 points) If you think $f(x)$ is a quadratic function $ax^2 + bx + c$, find a, b, and c. If you think $f(x)$ is an exponential function ab^x, find a and b. Your values do not have to be exact. Remember to show your work or explain your answers.

Based on the table and graph of f',

$$f'(x) = x + 1.2$$

Also, $f'(x) = 2ax + b$. So $2a = 1$, or $\boxed{a = 0.5}$, and $\boxed{b = 1.2}$. Also, $f(0) = a \cdot 0^2 + b \cdot 0 + c = c$. From the first table, $f(0) = 2.3$. So $\boxed{c = 2.3}$