MATH 124 Midterm 1

October 25, 2022

Name: ____________________

Student #: ____________________

Problem:	1	2	3	4	5	Total
Points:	15	15	15	8	7	60

INSTRUCTIONS:

- You have 80 minutes to take the test.
- There are 5 problems. Make sure you have all of them.
- Write your solution below the problem. There is scratch paper at the back of the test.
- The test is double-sided. Make sure you are reading the backs of pages!
- Unless otherwise stated, **show all your work for full credit**.
- Unless otherwise stated, all answers should be exact, without rounding.
- You are allowed to use one 8.5"×11" sheet of notes, front and back.
- You can use a TI-30X IIS calculator. No other calculator is allowed.

TIPS:

- The number of points a question is worth is not correlated to its difficulty.
- Don't spend too much time on one problem if you haven't looked at the rest of the test.
- There is partial credit. Even if you can't fully solve a problem, explaining your progress might get you a significant number of points.
- Make sure your calculator is in radians!!!

Good luck!

1. For each of these questions, find the limit. If the limit does not exist, state whether the limit is ∞ or $-\infty$ if either applies; if neither applies write "DNE".

   (a) (5 points) $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}$

   (b) (5 points) $\lim_{x \to \infty} \frac{3x^2 + x + 1}{7x^2 - x - 8}$

   (c) (5 points) $\lim_{x \to 0^+} \frac{\cos(\pi - x)}{x}$

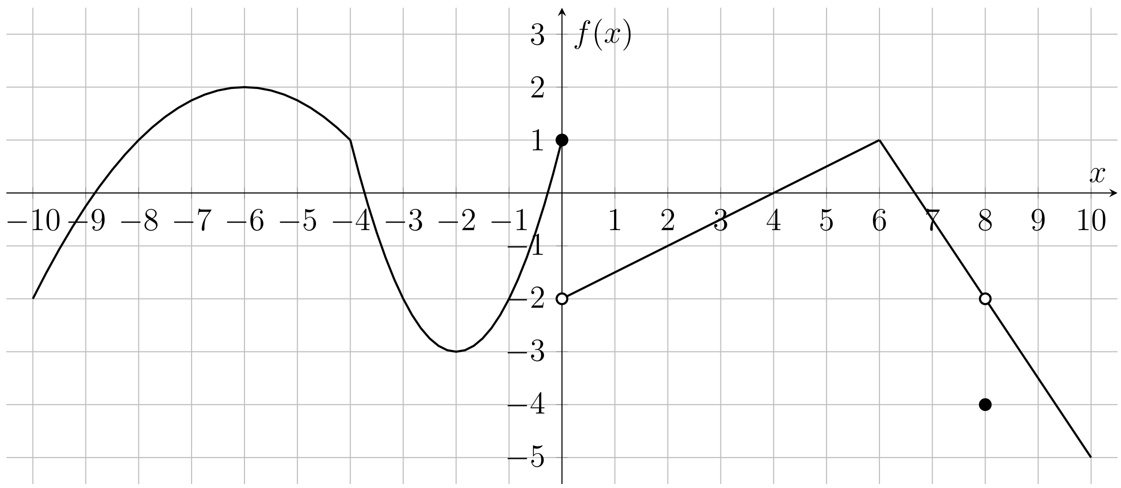
2. (a) (5 points) Let $f(x) = \sin x + \cos x - \sqrt{x} - \frac{1}{x}$. Find $f'(x)$.

(b) (5 points) Let $f(x) = (4x^2 + x)e^x$. Find $f'(x)$.

(c) (5 points) Find the equation of the tangent line to the graph of $y = x^3 - 3x - 1$ at the point $(2, 1)$.

3. For this problem, **you do not have to show your work**.

Answer the following questions based on the graph of $f(x)$ shown below. If a limit does not exist, state whether that limit is ∞ or $-\infty$ if either applies; if neither applies write "DNE".



(a) (1 point) $\lim_{x\to 0^-} f(x)$

(b) (1 point) $\lim_{x\to 0^+} f(x)$

(c) (1 point) $\lim_{x\to 0} f(x)$

(d) (2 points) $\lim_{x\to 8} f(x)$

(e) (2 points) List all values of a where $f'(a) = 0$.

(f) (2 points) List all values of a where $f'(a)$ is undefined.

(g) (2 points) $\lim_{h\to 0} \dfrac{f(2+h)+1}{h}$

(h) (2 points) $\lim_{x\to 8} f'(x)$

(i) (2 points) $\lim_{h\to 0^+} \dfrac{f(4+h)-2}{h}$ (Be careful!)

4. Let a be a number, and let

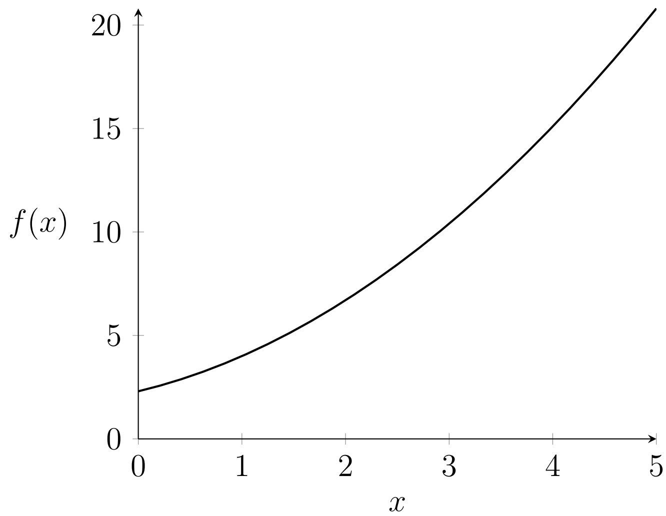
$$f(x) = \begin{cases} 3ax - 3 & \text{if } x < 1 \\ ax^2 + x & \text{if } x \geq 1 \end{cases}$$

(a) (4 points) Find all values of a which make $f(x)$ continuous at $x = 1$. If there are none, explain why.

(b) (4 points) Find all values of a which make $f(x)$ differentiable at $x = 1$. If there are none, explain why.

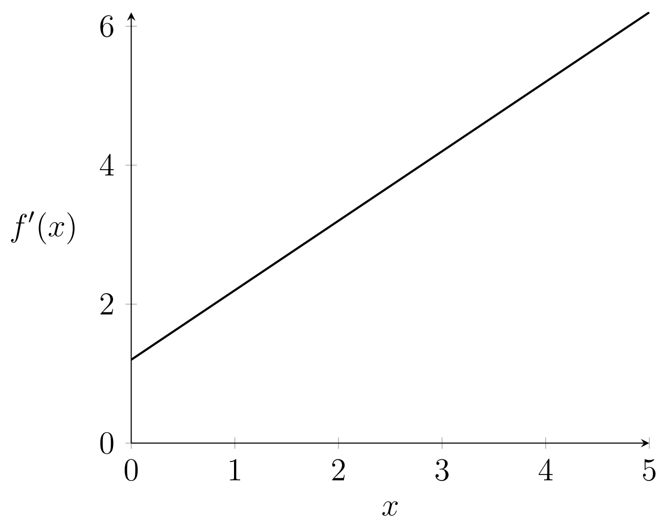
5. A scientist is studying a function $f(x)$. Through experiment, they have obtained a table of approximate values of $f(x)$ and an approximate graph of $f(x)$, shown below.

x	$f(x)$
0.0	2.3
0.5	3.0
1.0	4.0
1.5	5.2
2.0	6.7
2.5	8.4
3.0	10.4
3.5	12.6
4.0	15.1
4.5	17.8
5.0	20.8



Using a computer, the scientist also approximates the function $f'(x)$. The results are shown below.

x	$f'(x)$
0.0	1.2
0.5	1.7
1.0	2.2
1.5	2.7
2.0	3.2
2.5	3.7
3.0	4.2
3.5	4.7
4.0	5.2
4.5	5.7
5.0	6.2



(Problem 5 continued on next page.)

(a) (3 points) The scientist believes $f(x)$ is either a quadratic function (that is, of the form $f(x) = ax^2 + bx + c$ where a, b, and c are constants) or an exponential function (that is, of the form $f(x) = ab^x$ where a and b are constants). Based on the given information, which do you think is correct? Explain your answer.

Hint: Think about what the derivative of a quadratic function looks like, and what the derivative of an exponential function looks like.

(b) (4 points) If you think $f(x)$ is a quadratic function $ax^2 + bx + c$, find a, b, and c. If you think $f(x)$ is an exponential function ab^x, find a and b. Your values do not have to be exact. Remember to show your work or explain your answers.