

# Math 124B, Spring 2022 Midterm I

April 26, 2022

Name \_\_\_\_\_

Student Number \_\_\_\_\_

## Instructions.

- **These exams will be scanned. Please write your name and student number clearly for easy recognition.**
- There are 5 questions. The exam is out of 50 points.
- You are allowed to use one page of handwritten notes, 8.5 x 11, both sides ok.
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ( $\frac{2\ln 3}{\pi}$  and  $1/3$  are exact, 0.699 and 0.333 are approximations for the those numbers.)
- Each problem clearly states if you must show work. In cases where work is requested, **you may not get full credit for a right answer if your answer is not justified by your work.**

Question	points	Score
1	13	
2	12	
3	11	
4	8	
5	6	
Total	50	

1. (13 points) On this problem, place the answers you want graded in the provided boxes. If a box is left blank, you will get 0 points on that question. Give EXACT answers. Determine if the limit exists as a finite number or  $\pm\infty$  or DNE (does not exist). You only need to explain your answer in part (d)(iii); no work required for other parts. Some answers may involve unknown constants, as specified in each question.

(a)  $\lim_{h \rightarrow 5} \left( \frac{h-5}{h^2-25} \right) =$

(b)  $\lim_{h \rightarrow \infty} \left( \frac{h-5}{h^2-25} \right) =$

(c) If  $A$  and  $B$  are non-zero constants,

$$\lim_{t \rightarrow \frac{\pi}{2}} \left( \frac{\sqrt{\sin^2 t + A \cos^2 t} - \sin t}{B \cos^2 t} \right) =$$

(d) Consider the multipart function  $f(x) = \begin{cases} 3x^2 & \text{if } x \leq 0 \\ -4x^2 + 1 & \text{if } x > 0 \end{cases}$

(i)

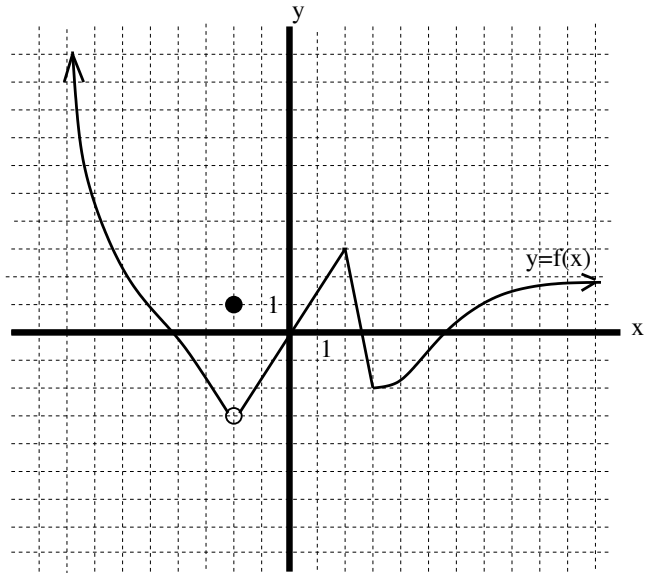
$$\lim_{h \rightarrow 0^-} f'(h) =$$

(ii)

$$\lim_{h \rightarrow 0^+} f'(h) =$$

(iii) Is  $f(x)$  differentiable at  $x = 0$ ? Explain.

2. (12 points) The graph of a function  $y = f(x)$  is pictured. The  $x$  and  $y$  axis are indicated and the dotted lines yield a grid with units of 1 in each direction. We will only grade your BOXED final answers. If you are asked to calculate a limit, determine if the limit exists as a finite number or  $\pm\infty$  or DNE (does not exist). No work required on this problem.



(a)  $\lim_{x \rightarrow -2} f(x) =$

(b)  $\lim_{x \rightarrow 0} \frac{f(x)}{x} =$

(c) Is  $y = \cos(\pi/x) \cdot f(x)$  continuous at  $x = -2$ ?

(d) What is the slope of the tangent line to the curve  $y = \frac{f(x)}{e^x}$  at  $x = -1$ ?

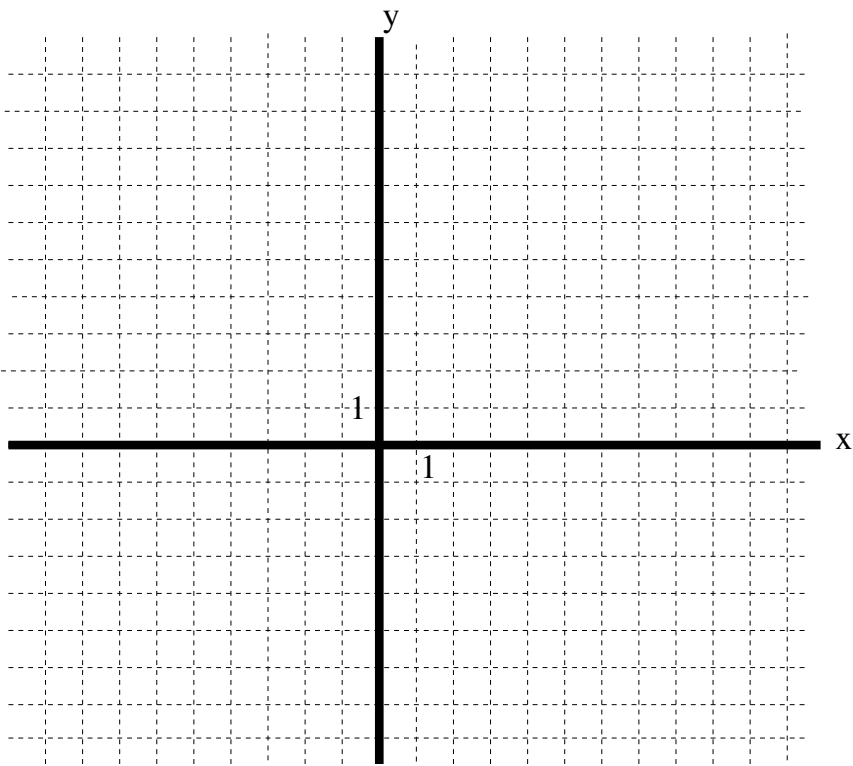
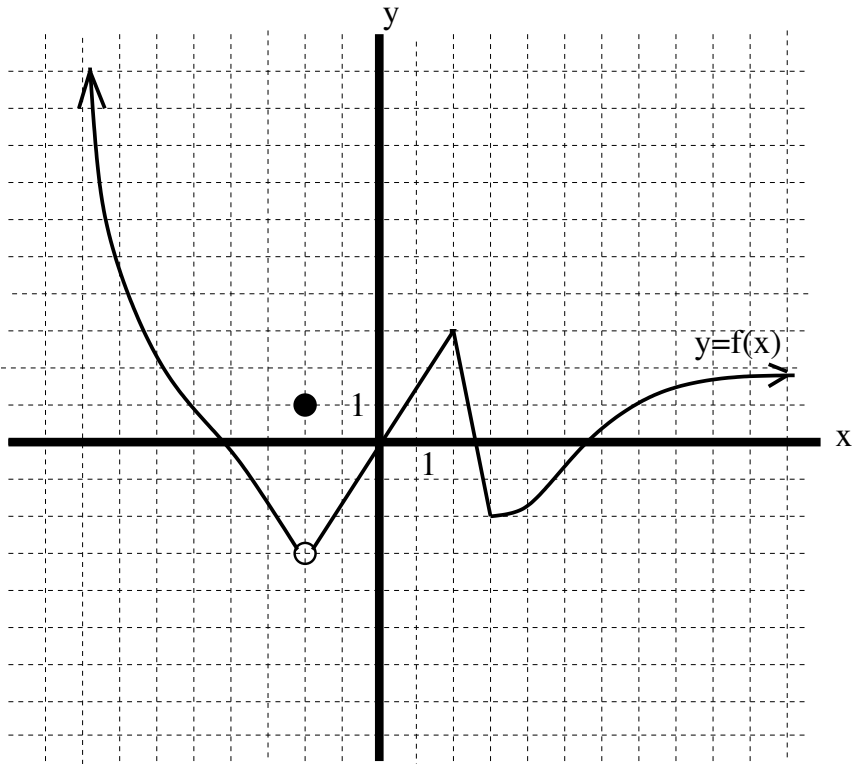
- (e) Circle the LARGEST number in this list:

$f(0)$

$f'(0)$

$f''(0)$

2. (continued). The graph of  $f(x)$  is reproduced here. Sketch the graph of  $y = f'(x)$  on the grid below:



3. (11 points) Let  $d(x) = \frac{1}{1+x^2}$ , then  $d'(x) = \frac{-2x}{(1+x^2)^2}$ . You must show work for parts (a) and (b) below.

(a) Find the equation of the tangent line to the graph of  $y = d(x)$  at the point  $P = (-1, 1/2)$ .

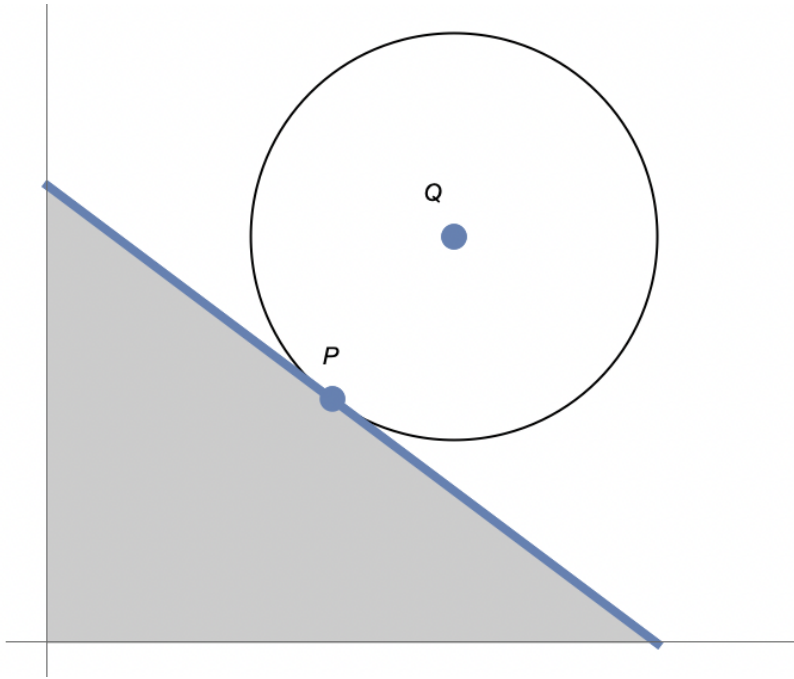
(b) Find the  $x$ -coordinates of ALL points on the graph of  $y = d(x)$  where the tangent line has  $x$ -intercept 2.

4. (8 points) An object moves along the  $x$ -axis and its location at time  $t \geq 0$  seconds is given by the function

$$x(t) = 10 - \frac{50t}{9 + t^2}$$

and units on the axis are “meters”. Assuming  $t \geq 0$ , where is the object located the first time the velocity is zero?

5. (6 points) In the picture below is a circle of radius 5 centered at  $Q = (10, 10)$ . We have drawn the line  $\ell$  tangent to the circle through the point  $P = (7, 6)$ . The line  $\ell$ , the positive  $x$ -axis and the positive  $y$ -axis determine a triangular region, as pictured. What is the area of the triangular region? ( You must explain how you arrived at your answer. No credit for answer only. )



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