Print Your Full Name

Solutions

Signature

Student ID Number

Quiz Section

Instructor's Name

TA's Name

Please read these instructions!

1. Your exam contains 9 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Unless otherwise stated, make sure to SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a box around your final answer to each question.
6. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π, $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course or from other courses.

Problem	Total Points
1	10
2	10
3	10
4	10
5	10
Total	50

1. (This problem contains parts (a)-(d).) Determine the following limits. If the limit is infinite, write ∞ or $-\infty$. If the limit does not exist and is not infinite, write DNE. Show all your work.

(a) (2 points) $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$

$$= \lim_{x\to 3} \frac{3-x}{3x(x-3)}$$

$$= \lim_{x\to 3} \frac{-1}{3x} = \boxed{-\frac{1}{9}}$$

+1 +1

(b) (3 points) $\lim_{x\to -\infty} \frac{7x-4}{\sqrt{x^2+2}}$

$$= \lim_{x\to -\infty} \frac{7-\frac{4}{x}}{\frac{\sqrt{x^2+2}}{x}}$$ +1

$$= \lim_{x\to -\infty} \frac{7-\frac{4}{x}}{\frac{\sqrt{x^2+2}}{-\sqrt{x^2}}}$$

$$= \lim_{x\to -\infty} \frac{7-\frac{4}{x}}{-\sqrt{1+\frac{2}{x^2}}} = \frac{7}{-\sqrt{1}} = \boxed{-7}$$

$\frac{4}{x} \to 0$, $\frac{2}{x^2} \to 0$

+1 +1

(c) (2 points) $\lim_{x \to -4^-} \frac{x^2 + 10x + 25}{x^2 + 9x + 20}$

$$= \lim_{x \to -4^-} \frac{(x+5)\cancel{(x+5)}}{(x+4)\cancel{(x+5)}}$$

+1

$$= \lim_{x \to -4^-} \frac{x+5 \to 1}{x+4} = \boxed{-\infty}$$

+1

$\to 0$ and neg.

(d) (3 points) $\lim_{t \to 1} \left(\frac{4}{(t-1)\sqrt{t}} - \frac{4}{t-1} \right)$

$$= \lim_{t \to 1} \frac{4 - 4\sqrt{t}}{(t-1)\sqrt{t}} \cdot \frac{4+4\sqrt{t}}{4+4\sqrt{t}}$$

+1

$$= \lim_{t \to 1} \frac{16 - 16t}{(t-1)\sqrt{t}\,(4+4\sqrt{t})}$$

$$= \lim_{t \to 1} \frac{16\cancel{(1-t)}}{-\cancel{(1-t)}\sqrt{t}\,(4+4\sqrt{t})}$$

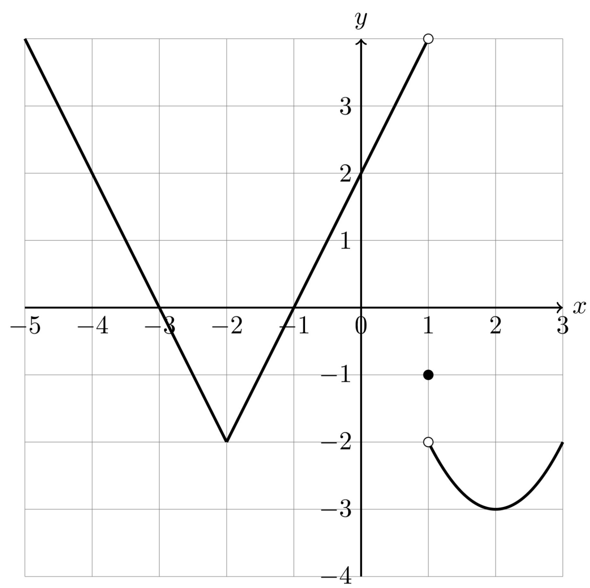
+1

$$= \lim_{t \to 1} \frac{16}{-\sqrt{t}\,(4+4\sqrt{t})} = \frac{16}{-1\,(4+4)} = \boxed{-2}$$

+1

2. **For this problem, you do not need to show your work.** Answer the following questions based on the graph of $y = f(x)$ below, which has the domain $-5 < x < 3$. For questions involving limits, if the limit is infinite, write ∞ or $-\infty$. If the limit does not exist, write DNE.

The function $y = f(x)$ is a line for $-5 < x < -2$; a line for $-2 < x < 1$; and a parabola for $1 < x < 3$.



(a) (1 point) $f(1) = \boxed{-1}$ (+1)

(b) (1 point) $f'(1) = \boxed{\text{DNE}}$ (+1)

(c) (1 point) $\lim_{x \to -2} f(x) = \boxed{-2}$ (+1)

(d) (1 point) $\lim_{x \to -2} f'(x) = \boxed{\text{DNE}}$ (+1)

(e) (1 point) $\lim_{x \to -4} \frac{f(x) - \overset{f(-4)}{2}}{\underset{x-(-4)}{x+4}} = f'(-4) = \boxed{-2}$ (+1)

(f) (1 point) $\lim_{h \to 0} \frac{f(2+h) + \overset{-f(2)}{3}}{h} = \boxed{0}$ (+1)

(g) (2 points) List the x-values for $-5 < x < 3$ where $y = f(x)$ ~~is NOT~~ continuous.

Your answer: $\boxed{1}$ (+2)

(h) (2 points) List the x-values for $-5 < x < 3$ where $y = f(x)$ ~~is NOT differentiable~~.

Your answer: $\boxed{-2, 1}$ (+1) (+1)

3. (10 points) Find $f'(x)$ for the function $f(x)$ below using a limit. No credit is awarded for use of other methods, such as derivative rules.

$$f(x) = 4x^2 - x$$

$$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2 - (x+h) - (4x^2 - x)}{h} \quad (+5)$$

$$= \lim_{h \to 0} \frac{4(x^2 + 2xh + h^2) - x - h - 4x^2 + x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{x} - h - \cancel{4x^2} + \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(8x + 4h - 1)}{\cancel{h}}$$

$$= \lim_{h \to 0} (8x + 4h - 1) = \boxed{8x - 1}$$

(+4) (+1)

4. Find the derivative of the following functions using derivative rules. You do not need to simplify your final answer.

(a) (3 points) $y = \dfrac{x^2 + 2e^x}{\sqrt{x} - 2}$

$$\boxed{y' = \frac{\underbrace{(\sqrt{x}-2)\cdot(2x+2e^x)}_{+1} - \underbrace{(x^2+2e^x)\cdot\left(\frac{1}{2\sqrt{x}}\right)}_{+1}}{\underbrace{(\sqrt{x}-2)^2}_{+1}}}$$

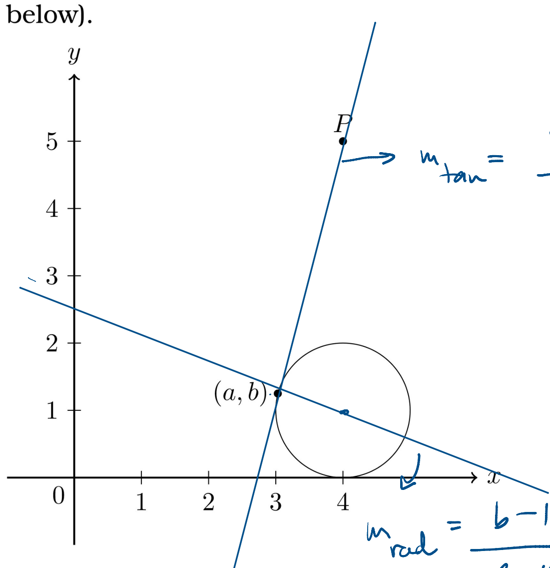
(b) (3 points) $y = 3 + 6e^{-7x}$

$$\boxed{y' = \underbrace{6}_{+1}\cdot\underbrace{-7}_{+1}\underbrace{e^{-7x}}_{+1}}$$

(c) (4 points) $y = 9x\cos(x)$

$$\boxed{y' = \underbrace{9x\cdot(-\sin x)}_{+2} + \underbrace{9\cdot\cos x}_{+2}}$$

5. (10 points) Let P be the point $(4, 5)$. Find the point (a, b) on the circle of radius one centered at $(4, 1)$ whose tangent line to the circle passes through P and has positive slope (see the figure below).



$\rightarrow m_{tan} = \dfrac{b-5}{a-4}$

$m_{rad} = \dfrac{b-1}{a-4}$

$m_{tan} = \dfrac{b-5}{a-4} = -\dfrac{a-4}{b-1}$ +2

$(b-5)(b-1) = -(a-4)^2$

circle equation: $(x-4)^2 + (y-1)^2 = 1$

+2 $(a-4)^2 + (b-1)^2 = 1$

$(a-4)^2 = 1 - (b-1)^2$

$(b-5)(b-1) = (b-1)^2 - 1$ +2

$\cancel{b^2} - 6b + 5 = \cancel{b^2} - 2b \cancel{+1} \cancel{-1}$

$-4b + 5 = 0$

$b = \dfrac{5}{4}$ +2

$(a-4)^2 = 1 - \left(\frac{5}{4} - 1\right)^2$

$(a-4)^2 = 1 - \left(\frac{1}{4}\right)^2$

$(a-4)^2 = \dfrac{15}{16}$

$a - 4 = \pm \dfrac{\sqrt{15}}{4}$

$a = 4 \pm \dfrac{\sqrt{15}}{4}$

Since $a < 4 \Rightarrow a = 4 - \dfrac{\sqrt{15}}{4}$

$\boxed{\text{Point: } \left(4 - \frac{\sqrt{15}}{4}, \frac{5}{4}\right)}$

+2

Extra scratch paper.

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