# Math 124 B and D Midterm 1

**Prof. Charles Camacho** **Math 124 B & D** **Midterm 1 Exam, Winter 2024**

| Print Your Full Name | Signature |
|---|---|
| Solutions | |
| **Student ID Number** | **Quiz Section** |
| | |
| **Instructor's Name** | **TA's Name** |
| | |

**Please read these instructions!**

1. Your midterm exam contains 9 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The entire midterm exam is worth 54 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Make sure to ALWAYS SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a box around your final answer to each question.
6. There is plenty of space on the exam to do your work. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, $3\pi$, $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations $9.424778$, $1.4142$, and $0.693147$ are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course.

| Problem | Total Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 10 | |
| 3 | 12 | |
| 4 | 10 | |
| 5 | 10 | |
| Total | 54 | |

1. (This problem contains parts (a)-(d).) Determine the following limits. If the limit is infinite, write $\infty$ or $-\infty$. If the limit does not exist and is not infinite, write DNE. Show all your work.

(a) (3 points) $\lim_{x\to 1} \dfrac{e^{3x^2-1}}{x+1}$

$= \dfrac{e^{3(1)^2-1}}{1+1} = \boxed{\dfrac{e^2}{2}}$

(b) (3 points) $\lim_{x\to 2} \left(\dfrac{1}{x-2} - \dfrac{4}{x^2-4}\right)$

$= \lim_{x\to 2} \left( \dfrac{1}{x-2} - \dfrac{4}{(x-2)(x+2)} \right)$

$= \lim_{x\to 2} \dfrac{x+2-4}{(x-2)(x+2)}$

$= \lim_{x\to 2} \dfrac{x-2}{(x-2)(x+2)}$

$= \lim_{x\to 2} \dfrac{1}{x+2}$

$= \boxed{\dfrac{1}{4}}$

(c) (3 points) $\lim_{x\to\infty} \dfrac{2x+3}{\sqrt{x^2-2x+3}}$

$$= \lim_{x\to\infty} \frac{\frac{2x+3}{x}}{\frac{\sqrt{x^2-2x+3}}{x}}$$

$$= \lim_{x\to\infty} \frac{2+\frac{3}{x}}{\frac{\sqrt{x^2-2x+3}}{\sqrt{x^2}}}$$

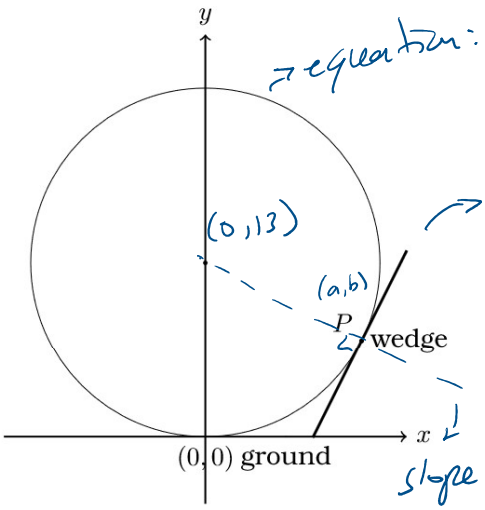
$$= \lim_{x\to\infty} \frac{2+\left(\frac{3}{x}\right)\to 0}{\sqrt{1-\left(\frac{2}{x}\right)+\left(\frac{3}{x^2}\right)}} = \boxed{2}$$

(d) (3 points) $\lim_{x\to 0^-} \arctan\left(\dfrac{1}{x}\right)$

$$= \arctan\left(\lim_{x\to 0^-} \frac{1}{x}\right)$$

$$= \boxed{-\frac{\pi}{2}} \quad \text{since } \frac{1}{x} \to -\infty \text{ as } x\to 0^-$$

2. (10 points) Sometimes, Prof. Camacho parks his bicycle outside Padelford Hall, and he notices that his bike tires touch an angled wedge of concrete. If his circular tires have a radius of 13 inches and the slope that the wedge makes with the ground (represented by the $x$-axis) is 2, find the coordinates of the point $P = (a, b)$ where the tire hits the wedge. (See the figure below.)




→ equation: $x^2 + (y-13)^2 = 13^2$

→ slope $= 2 = -\frac{a}{b-13}$

$2(b-13) = -a$

$a^2 + (b-13)^2 = 13^2$

slope $= \frac{b-13}{a}$

$4(b-13)^2 + (b-13)^2 = 13^2$

$5(b-13)^2 = 13^2$

$(b-13)^2 = \frac{13^2}{5}$

$b - 13 = \pm \frac{13}{\sqrt{5}} \rightarrow b = \frac{\pm 13}{\sqrt{5}} + 13$

$\Rightarrow a = -2(b-13) = -2\left(\pm \frac{13}{\sqrt{5}}\right)$ → choose − from ≥

$= \frac{2 \cdot 13}{\sqrt{5}} = \frac{26}{\sqrt{5}}$

$b = -\frac{13}{\sqrt{5}} + 13$

$$P = \left(\frac{26}{\sqrt{5}}, 13 - \frac{13}{\sqrt{5}}\right)$$

3. Find an equation of the tangent line to the following functions at the given points.

(a) (4 points) $y = x^3 - 3x + 1, \quad (2, 3)$

$y' = 3x^2 - 3$

$y'(2) = 3 \cdot 2^2 - 3$

$= 9$

$\Rightarrow \boxed{y = 9(x-2)+3}$

(b) (4 points) $y = e^x(\tan(x) - x), \quad (0, 0)$

$y' = e^x(\sec^2 x - 1) + (\tan x - x) \cdot e^x$

$y'(0) = 1(1-1) + (0-0) \cdot e^0 = 0$

$\Rightarrow \boxed{y = 0}$

(c) (4 points) $y = \dfrac{1+x}{1+e^x}, \quad (0, 1/2)$

$y' = \dfrac{(1+e^x) \cdot 1 - (1+x) \cdot e^x}{(1+e^x)^2}$

$y'(0) = \dfrac{(1+1) \cdot 1 - (1+0) \cdot 1}{(1+1)^2} = \dfrac{2-1}{4} = \dfrac{1}{4}$

$\boxed{y = \frac{1}{4}x + \frac{1}{2}}$

4. (10 points) Find the derivative function of the following function **using the limit definition of the derivative**. You must calculate the derivative using limits in order to receive credit.

$$f(x) = \frac{2}{3x^2}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{3(x+h)^2} - \frac{2}{3x^2}}{h}$$

$$= \lim_{h\to 0} \frac{2\cdot 3x^2 - 2\cdot 3(x+h)^2}{h\cdot 3(x+h)^2\cdot 3x^2}$$

$$= \lim_{h\to 0} \frac{6x^2 - 6(x^2+2xh+h^2)}{9h(x+h)^2\cdot x^2}$$

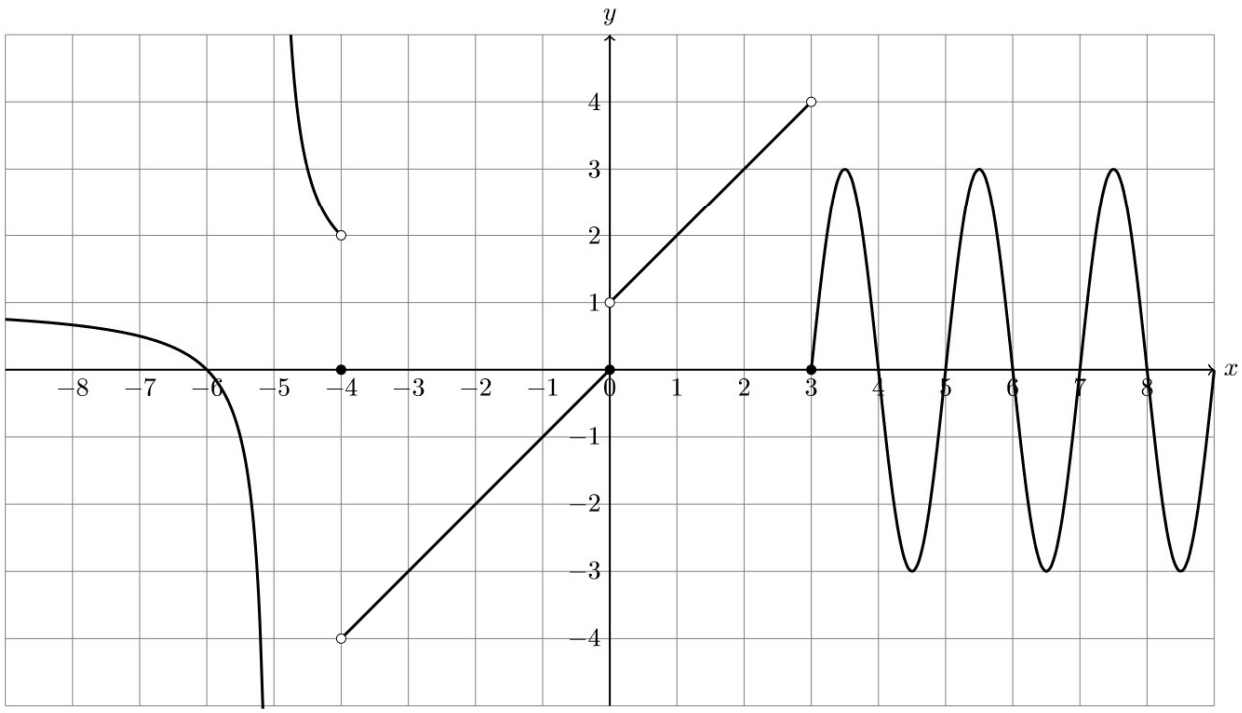
$$= \lim_{h\to 0} \frac{6x^2 - 6x^2 - 12xh - 6h^2}{9h(x+h)^2\cdot x^2}$$

$$= \lim_{h\to 0} \frac{-12x - 6h}{9(x+h)^2\cdot x^2}$$

$$= \frac{-12x}{9\cdot x^4} = \boxed{\frac{-4}{3x^3}}$$

5. **For this problem, you do not need to show your work.** Answer the following questions based on the graph of $y = f(x)$ below, which has the domain $-\infty < x < \infty$ except at $x = -5$. For questions involving limits, if the limit is infinite, write $\infty$ or $-\infty$. If the limit does not exist, write DNE.

   The function $y = f(x)$ is a shifted graph of $y = (1/x)+1$ for $-\infty < x < -4$; a line between $-4 < x \leq 0$; a line between $0 < x < 3$; and a sinusoidal curve between $3 \leq x < \infty$.




(a) (1 point) $\lim_{x \to -4} f(x) =$ DNE

(b) (1 point) $f(-4) =$ 0

(c) (1 point) $\lim_{x \to 0} f'(x) =$ 1

(d) (1 point) $f'(0) =$ DNE

(e) (1 point) $\lim_{x \to 2} \dfrac{f(x) - 3}{x - 2} =$ 1

(f) (1 point) $\lim_{h \to 0} \dfrac{f(3.5 + h) - 3}{h} =$ 0

(g) (1 point) $\lim_{x \to \infty} f(x) =$ DNE

(h) (1 point) $\lim_{x \to 0^-} \dfrac{f(x)}{x} =$ 1

(i) (1 point) List the $x$-values for $-\infty < x < \infty$ where $y = f(x)$ is NOT continuous.

Your answer: -5, -4, 0, 3

(j) (1 point) List the $x$-values for $-\infty < x < \infty$ where $y = f(x)$ is NOT differentiable.

Your answer: -5, -4, 0, 3

**Extra scratch paper.**

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