Math 124 B C Midterm 1

Print Your Name

Solutions

Signature

Student ID Number

Quiz Section

Instructor's Name

TA's Name

Please read these instructions!

1. Your midterm exam contains 10 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The entire midterm exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Make sure to ALWAYS SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Partial credit is awarded as earned. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a $\boxed{\text{box around your final answer}}$ to each question.
6. There is plenty of space on the exam to do your work. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π, $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course.

Problem	Total Points	Score
1	20	
2	15	
3	20	
4	15	
5	15	
6	~~20~~ 15	
Total	100	

1. (This problem contains parts (a)-(d).) Determine the following limits. If the limit is infinite, write ∞ or $-\infty$. If the limit does not exist, write DNE. Show all your work.

(a) (5 points) $\lim_{x\to -1} \frac{2x^2+3x+1}{x^2-2x-3}$

$$= \lim_{x\to -1} \frac{(2x+1)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$$

$$= \lim_{x\to -1} \frac{2x+1}{x-3} = \frac{2(-1)+1}{-1-3} = \frac{-1}{-4} = \boxed{\frac{1}{4}}$$

(b) (5 points) $\lim_{x\to 3^-} \frac{1}{-(2x-6)^3} = \boxed{\infty}$

0 and negative

→ 0 and positive

(c) (5 points) $\lim_{x\to-\infty} \frac{4x-1}{\sqrt{x^2+2x}}$

$$= \lim_{x\to-\infty} \frac{\frac{4x-1}{x}}{\frac{\sqrt{x^2+2x}}{x}} \qquad x = -\sqrt{x^2} \text{ since } x<0$$

$$= \lim_{x\to-\infty} \frac{4 - \frac{1}{x} \to 0}{-\sqrt{1+\frac{2}{x}} \to 0}$$

$$= \boxed{-4}$$

(d) (5 points) $\lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x^2-2x} \cdot \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$

$$= \lim_{x\to 2} \frac{x^2+5-9}{(x^2-2x)(\sqrt{x^2+5}+3)}$$

$$= \lim_{x\to 2} \frac{x^2-4}{x(x-2)(\sqrt{x^2+5}+3)}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{x\cancel{(x-2)}(\sqrt{x^2+5}+3)}$$

$$= \lim_{x\to 2} \frac{x+2}{x(\sqrt{x^2+5}+3)} = \frac{4}{2(\sqrt{9}+3)} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

2. (15 points) Find the derivative function of the following function using the limit definition of the derivative. You must calculate the derivative using limits in order to receive credit.

$$f(x) = \frac{1}{\sqrt{2x}}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{2(x+h)}} - \frac{1}{\sqrt{2x}}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2x} - \sqrt{2(x+h)}}{h\sqrt{2(x+h)} \cdot \sqrt{2x}} \cdot \frac{\sqrt{2x} + \sqrt{2(x+h)}}{\sqrt{2x} + \sqrt{2(x+h)}}$$

$$= \lim_{h \to 0} \frac{2x - 2(x+h)}{h \cdot \sqrt{2(x+h)} \cdot \sqrt{2x}\left(\sqrt{2x} + \sqrt{2(x+h)}\right)}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}\sqrt{2(x+h)}\sqrt{2x}\left(\sqrt{2x} + \sqrt{2(x+h)}\right)}$$

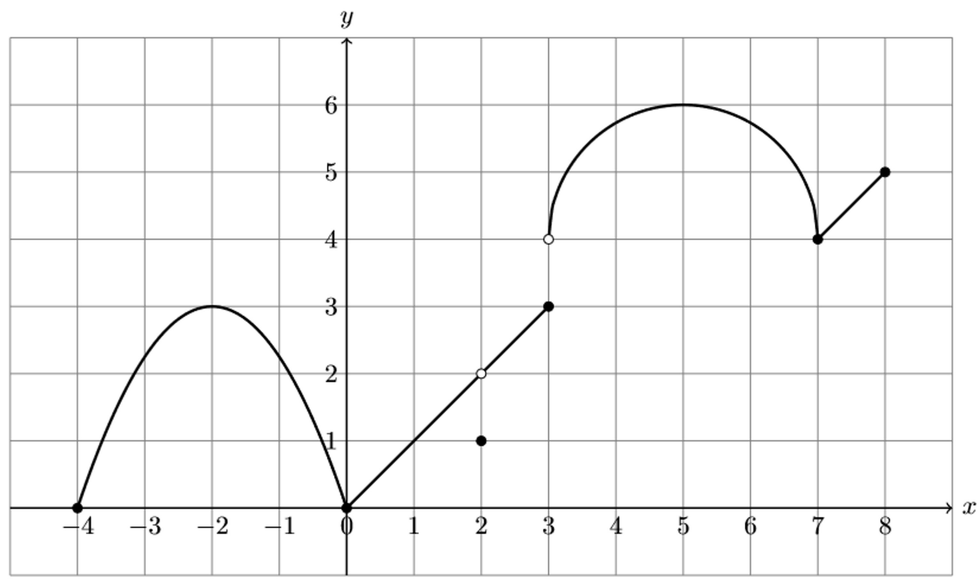
$$= \lim_{h \to 0} \frac{-2}{\sqrt{2(x+h)} \cdot \sqrt{2x}\left(\sqrt{2x} + \sqrt{2(x+h)}\right)}$$

$$= \frac{-\cancel{2}}{2x \cdot \cancel{2} \cdot \sqrt{2x}}$$

$$= \boxed{-\frac{1}{2 \cdot \sqrt{2} \cdot x^{3/2}}}$$

3. **For this problem, you do not need to show your work.** Answer the following questions based on the graph of $y = f(x)$ below, which has the domain $-4 \le x \le 8$. For questions involving limits, if the limit is infinite, write ∞ or $-\infty$. If the limit does not exist, write DNE.

The function $y = f(x)$ is a parabola for $-4 \le x \le 0$; a line between $0 \le x < 3$; a semicircle for $-3 < x \le 7$; and a line between $7 \le x \le 8$.



(a) (2 points) $\lim_{x \to 2} f(x) =$ 2

(b) (2 points) $f(2) =$ 1

(c) (2 points) $f'(2) =$ DNE

(d) (2 points) $\lim_{x \to 7^+} \frac{f(x) - 4}{x - 7} =$ 1

(e) (2 points) $\lim_{x \to 7^-} \frac{f(x) - 4}{x - 7} =$ $-\infty$

(f) (1 point) True or False: $y = f(x)$ is left-continuous at $x = 3$.

Your answer: True

(g) (1 point) True or False: $y = f(x)$ is left-continuous for $-4 < x < 8$.

Your answer: False

(h) (2 points) $\lim_{h \to 0} \frac{f(1 + h) - 1}{h} =$ 1

(i) (2 points) List the x-values for which $f'(x) = 0$.

Your answer: −2, 5

(j) (2 points) List the x-values for $-4 < x < 8$ where $y = f(x)$ is NOT continuous.

Your answer: 2, 3

(k) (2 points) List the x-values for $-4 < x < 8$ where $y = f(x)$ is NOT differentiable.

Your answer: 0, 2, 3, 7

4. (a) (5 points) Find the derivative of $y = (5x^3 - \sqrt{5}x^2 - 20x + 13)(\sin x - \cos x)$.

$$\boxed{y' = (5x^3 - \sqrt{5}x^2 - 20x + 13)(\cos x + \sin x) + (\sin x - \cos x)(15x^2 - 2\sqrt{5}x - 20)}$$

(b) (5 points) Find the derivative of $y = \dfrac{3x-2}{2x+5}$.

$$\boxed{y' = \frac{(2x+5)\cdot 3 - (3x-2)\cdot 2}{(2x+5)^2}}$$

(c) (5 points) Find an equation of the tangent line to the graph of the function $y = 4x^3 - 7x^2$ at the point corresponding to $x = 2$.

$y' = 12x^2 - 14x \rightarrow$ slope $= 12(2)^2 - 14(2)$
$= 48 - 28 = 20$

Point $\rightarrow (2, 4(2)^3 - 7(2)^2) = (2, 4)$
$32 - 28$

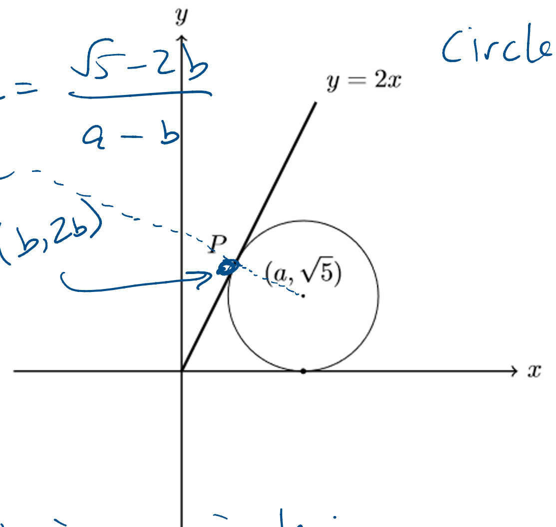
$\Rightarrow$ Tangent line equation : $\boxed{y = 20(x-2) + 4}$

5. (15 points) Andy would like to plant a tree with a circular pot in a section of their garden bounded by two fences. Andy models this section in the xy-plane by using the lines $y = 2x$ and the x-axis to represent the fences. (See the figure below.) The tree is to be planted in the corner so that the tree's circular pot is as close to both fences as possible. Andy measured the radius of the tree's pot to be $\sqrt{5}$ and knows the y-coordinate of the center of the circle to be $\sqrt{5}$. Find the x-coordinate of the center of the pot, labeled a below.

[Hint: Let the x-coordinate of the point P below be $x = b$, so that the y-coordinate is $y = 2b$. Then use the circle's equation $(x - a)^2 + (y - \sqrt{5})^2 = 5$ and the fact that $2b > \sqrt{5}$.]

$\text{slope} = \dfrac{\sqrt{5}-2b}{a-b}$

$(b, 2b)$



Circle Equation: $(x-a)^2+(y-\sqrt{5})^2=5$

$$2 = -\frac{1}{\frac{\sqrt{5}-2b}{a-b}}$$

$$2 = \frac{b-a}{\sqrt{5}-2b}$$

$$\Rightarrow 2(\sqrt{5}-2b) = b-a$$

Point is on circle:

$$\Rightarrow (b-a)^2+(2b-\sqrt{5})^2=5$$

$$\left(2(\sqrt{5}-2b)\right)^2+(2b-\sqrt{5})^2=5$$

$$4(\sqrt{5}-2b)^2+(\sqrt{5}-2b)^2=5$$

$$5(\sqrt{5}-2b)^2=5$$

$$(\sqrt{5}-2b)^2=1$$

$$\sqrt{5}-2b=\pm 1$$

$$\sqrt{5}\mp 1=2b$$

$$\sqrt{5}+1=2b \quad (\text{since } 2b>\sqrt{5})$$

$$b=\frac{1+\sqrt{5}}{2}$$

$$\Rightarrow 2\left(\sqrt{5}-(\sqrt{5}+1)\right)=\frac{1+\sqrt{5}}{2}-a$$

$$2(-1)-\frac{1+\sqrt{5}}{2}=-a$$

$$a=2+\frac{1+\sqrt{5}}{2}$$

$$\boxed{a=\frac{5+\sqrt{5}}{2}}$$

6. (~~20~~ 15 points) Find the values of a and b that make the function below continuous everywhere.

$$f(x) = \begin{cases} 3x^2 - 1 & \text{if } x < 0 \\ ax + b & \text{if } 0 \le x \le 1 \\ \sqrt{x+8} & \text{if } x > 1 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$3(0)^2 - 1 = a(0) + b$$

$$\boxed{-1 = b}$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$a(1) + b = \sqrt{1+8}$$

$$a - 1 = \sqrt{9}$$

$$\boxed{a = 4}$$

Extra scratch paper.

Extra scratch paper.