Solutions to Math 124 C Fall 2025 Midterm I

1. (a) $f'(x) = \dfrac{(2xe^x + x^2e^x)(x^5+3) - (x^2e^x)(5x^4)}{(x^5+3)^3}$

   (b) $f'(x) = 3x^2\cos\left(x^3\right) - \dfrac{2x}{(x^2+1)^2}$

   (c) $f'(x) = e^{4x}\tan(x) + 4xe^{4x}\tan(x) + xe^{4x}\sec^2(x)$

2. (a)

$$\lim_{x\to 3^+} \frac{4x^2+1}{x^2-9} = \infty \qquad \lim_{x\to -3^+} \frac{4x^2+1}{x^2-9} = -\infty \qquad \lim_{x\to \infty} \frac{4x^2+1}{x^2-9} = 4$$

$$\lim_{x\to 3^-} \frac{4x^2+1}{x^2-9} = -\infty \qquad \lim_{x\to -3^-} \frac{4x^2+1}{x^2-9} = \infty \qquad \lim_{x\to -\infty} \frac{4x^2+1}{x^2-9} = 4$$

   (b) D

   (c) $\displaystyle\lim_{x\to 2} \frac{x^2-5x+6}{2x-4} = \lim_{x\to 2} \frac{(x-2)(x-3)}{2(x-2)} = \lim_{x\to 2} \frac{x-3}{2} = -\frac{1}{2}$

   (d) Note that this is $f'(x)$ where $f(x) = \frac{1}{x+3}$ so $f'(x) = -\frac{1}{(x+3)^2}$. It can also be evaluated algebraically cleaning up fractions.

3. The parabola in the picture below has the equation

$$y = \frac{1}{8}x^2 - 3x + \frac{231}{8}.$$

The circle has center at $(0, 7)$. The line is tangent to the parabola at point R and the circle at point P. Point R is at $x = 15$.

   (a) When $x = 15$,

$$y(15) = \frac{15^2}{8} - 3(15) + \frac{231}{8} = 12$$

   the derivative is

$$y' = \frac{2x}{8} - 3 = \frac{x}{4} - 3$$

   so the slope is $m = \frac{15}{4} - 3 = \frac{3}{4}$ so the tangent line is

$$y - 12 = \frac{3}{4}(x - 15)$$

   or $y = \frac{3}{4}x + \frac{3}{4}$.

   (b) If $P(x, y)$, since the tangent line on a circle is always perpendicular to the radial line we have

$$\frac{y-7}{x-0} \cdot \frac{3}{4} = -1$$

   so $4x + 3y = 21$. The point P is also on the tangent line so $y = \frac{3}{4}x + \frac{3}{4}$. Solving the two equations (by substitution or elimination) we get $x = y = 3$.

   (c) Distance formula:

$$\sqrt{(0-3)^2 + (7-3)^2} = 5.$$

4. (a) $x = -5$
   (b) $x = -5, 1, 8$
   (c) $(-2, 1)$ and $(4.5, 9)$
   (d)
$$\frac{f(4.5) - f(-2)}{4.5 - (-2)} = -\frac{16}{13}$$
   (e) $\lim_{x \to -5^-} f(x) = 0$
   (f) $f'(-2) = 0$
   (g) $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$ DNE (left and right limits don't match)
   (h) $\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} \approx -8/3$