Math 124 C Fall 2025 Midterm I

October 21, 2025

Name_____

Student Number_____

Instructions

- These exams will be scanned. **Please write your name and student number clearly and do not write very close to the edges of the paper.**

- There are 4 questions. The exam is out of 50 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. No printed material allowed. **Hand in your notes with your exam.**

- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and $1/3$ are exact, 0.699 and 0.333 are approximations for those numbers.)

- Show your work. If we cannot read or follow your work, we cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.**

1. (4 points each) Differentiate the following functions. You do not need to simplify your answers.

   (a) $f(x) = \dfrac{x^2 e^x}{x^5 + 3}$

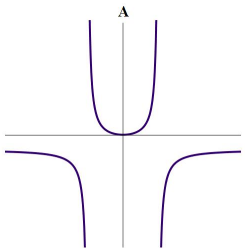
   (b) $f(x) = \sin\left(x^3\right) + \dfrac{1}{x^2 + 1}$

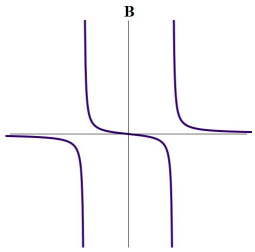
   (c) $f(x) = xe^{4x} \tan(x)$

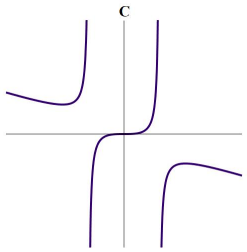
2. (a) (6 points) Find the following limits. You do not have to show your work.

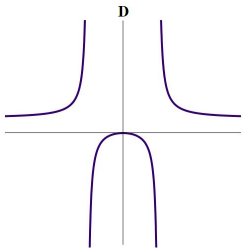
$$\lim_{x\to 3^+} \frac{4x^2+1}{x^2-9} = \qquad \lim_{x\to -3^+} \frac{4x^2+1}{x^2-9} = \qquad \lim_{x\to \infty} \frac{4x^2+1}{x^2-9} =$$

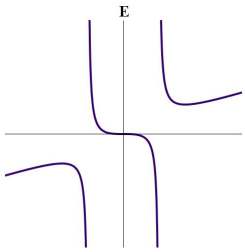
$$\lim_{x\to 3^-} \frac{4x^2+1}{x^2-9} = \qquad \lim_{x\to -3^-} \frac{4x^2+1}{x^2-9} = \qquad \lim_{x\to -\infty} \frac{4x^2+1}{x^2-9} =$$

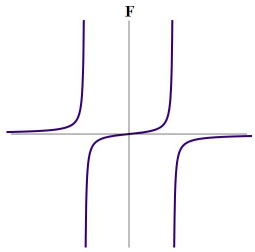












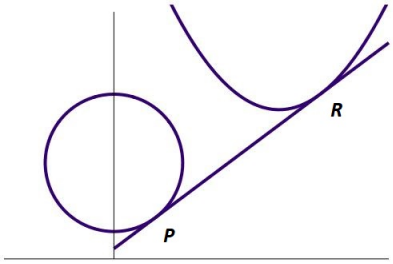
(b) (1 point) From the limits in part (a), the graph of $y = \frac{4x^2+1}{x^2-9}$ must be given by picture $\square$.

(c) (3 points) $\lim_{x\to 2} \dfrac{x^2-5x+6}{2x-4} =$

(d) (3 points) $\lim_{h\to 0} \dfrac{\frac{1}{x+3+h} - \frac{1}{x+3}}{h} =$

3. The parabola in the picture below has the equation

$$y = \frac{1}{8}x^2 - 3x + \frac{231}{8}.$$



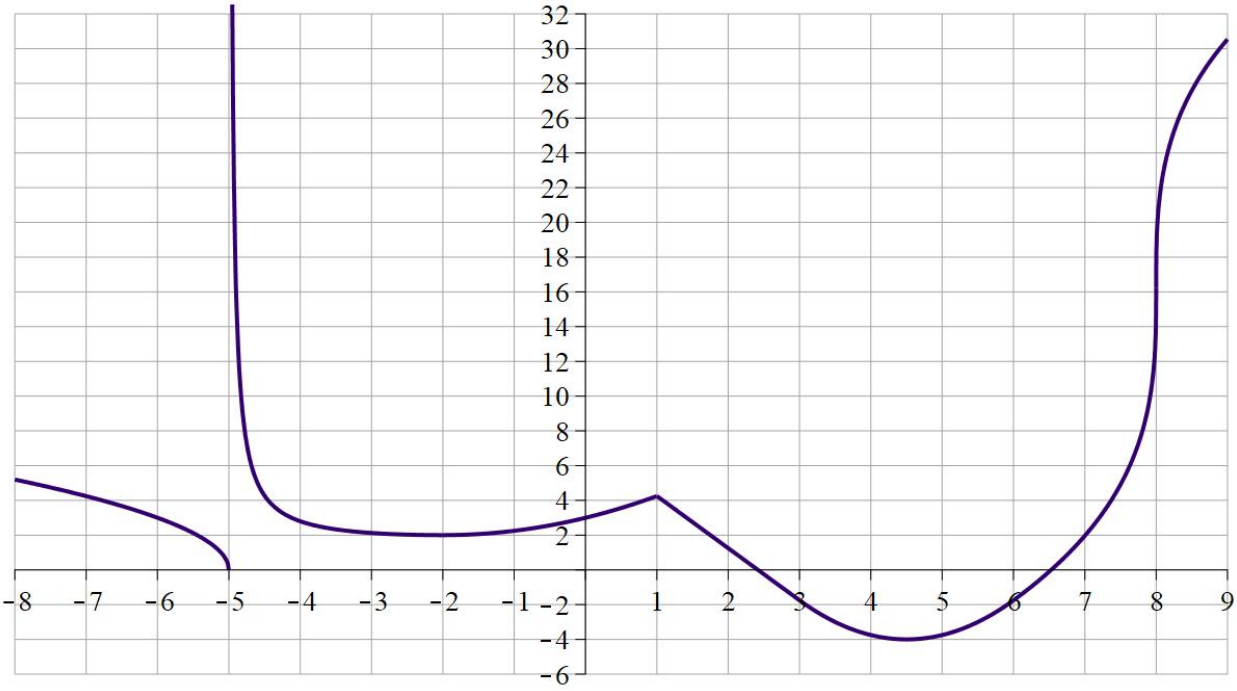
The circle has center at $(0, 7)$. The line is tangent to the parabola at point R and the circle at point P. Point R is at $x = 15$.

(a) (6 points) Find the equation of the common tangent line. Give your answer in the form $y = mx + b$.

(b) (6 points) Find the coordinates of the point P.

(c) (2 points) Find the radius of the circle.

4. (11 points) The function $y = f(x)$ has domain $-8 \leq x \leq 9$. Use the graph of $y = f(x)$ below to answer the questions. You do not need to explain your answers. If you cannot see the value of the function at a point exactly, make your best estimate. This will be taken into account while grading.



(a) List the values of x where the function is not continuous.

(b) List the values of x where the function is not differentiable.

(c) List the intervals where $f'(x) > 0$.

(d) Compute the average rate of change of $f(x)$ from $x = -2$ to $x = 4.5$.

(e) $\lim_{x \to -5^-} f(x) =$

(f) $f'(-2) =$

(g) $\lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h} =$

(h) $\lim_{h \to 0^+} \dfrac{f(1+h) - f(1)}{h} =$

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