Math 124 (Pezzoli)
Winter 2017
Midterm #1 (60 points)

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Name ______________

TA: ______________

Section: ______________

Instructions:

- Your exam contains 5 problems. The entire exam is worth 60 points.
- Your exam should contain 7 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides).
- The only calculator allowed is TI-30x IIS

Problem #1 (15 pts) ______

Problem #2 (10 pts) ______

Problem #3 (15 pts) ______

Problem #4 (10 pts) ______

Problem #5 (10 pts) ______

TOTAL (60 pts) ______

1. Calculate the following limits. Your answer should be either a number, or $+\infty$, or $-\infty$ or DNE (does not exist). Make sure to justify all steps.

(a) (5 points) $\lim_{x\to 1} \dfrac{x^2-1}{x^2-2x+1}$

$$\frac{(x-1)(x+1)}{(x-1)^2} = \frac{x+1}{x-1}$$

$$\lim_{x\to 1^+} \frac{x+1}{x-1} \begin{matrix}\to 2 \\ \to 0^+\end{matrix} = +\infty \qquad \lim_{x\to 1^-} \frac{x+1}{x-1} \begin{matrix}\to 2 \\ \to 0^-\end{matrix} = -\infty$$

so $\lim_{x\to 1} \dfrac{x^2-1}{x^2-2x+1}$ $\boxed{\text{DNE}}$

(b) (5 points) $\lim_{x\to 0} 1 + x^2 \cdot \sin\dfrac{1}{x} =$

$$-1 \le \sin\frac{1}{x} \le 1$$

$$-x^2 \le x^2 \sin\frac{1}{x} \le x^2 \qquad \lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$$

so by the squeeze th $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$ and

$$\lim_{x\to 0} 1 + x^2 \sin\frac{1}{x} = \boxed{1}$$

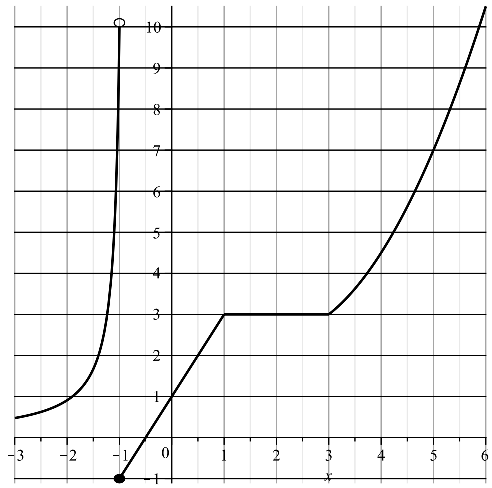
(c) (5 points) Find all values of the constant parameter A such that

$$\lim_{x\to +\infty} \frac{Ax^2-3}{7x^2-3x+2} = 2$$

$$\frac{x^2\,(A - 3/x^2)}{x^2\,(7 - 3/x + 2/x^2)} \begin{matrix}\to A \\ \to 7\end{matrix} \qquad \text{so we want } \frac{A}{7} = 2$$

$$\boxed{A = 14}$$

2. Below is the graph of a function $y = f(x)$.



(a) (3 points) List all values a in the interval $[-2, 5]$ such that f is not continuous at a. No justification necessary.

$a = -1$

(b) (4 points) List all values a in the interval $[-2, 5]$ such that f is not differentiable at a. No justification necessary.

$a = -1, 1, 3$

(c) (3 points) Calculate $\lim_{h \to 0} \dfrac{f(h) - 1}{h}$. Show your work.

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = f'(0)$$

graphically $f'(0) = \boxed{2}$

3. The position with respect to a certain origin O of a certain object, that we will call object 1, moving along a straight line, is given by $s_1(t) = t^2 - t + 2$. t is measured in minutes , s_1 in feet.

   (a) (3 points) Find the average velocity of the object in the time interval $[3, 4]$

$$\frac{s_1(4) - s_1(3)}{4-3} = (16-4+2)-(9-3+2) = \boxed{6}\ \text{feet}/\text{min}$$

   (b) (3 points) Find the instantaneous velocity of the object at t=4.

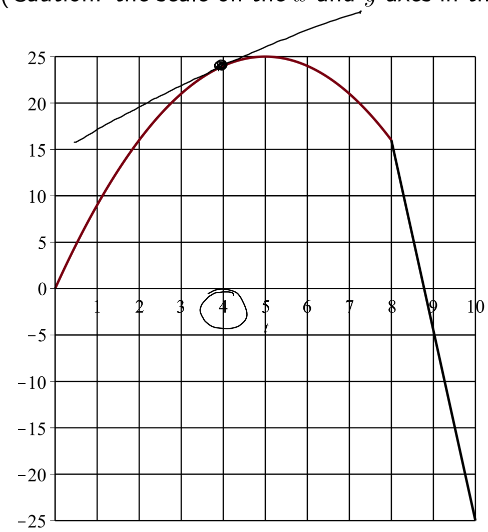
$$v(t) = s_1'(t) = 2t-1$$
$$v(4) = 8-1 = \boxed{7}\ \text{feet}/\text{min}$$

   (c) (3 points) Find the acceleration $a(t)$ of the object

$$a(t) = v'(t) = 2\ \text{ft}/\text{min}^2$$

(PROBLEM CONTINUED)

A different object, that we will call object 2, moves along the same line and its position $s_2(t)$ is given by the function in the graph below.Time t is still measured in minutes, and distances s_2 in feet (Caution: the scale on the x and y axes in the graph below is different)



(d) (3 points) Estimate which object, object 1 or object 2, is moving faster at time $t = 4$. Remember to justify your answer.

On the graph we can estimate that the slope of tangent line at $t=4$ is less than 7 so

$\boxed{\text{object 1 is faster}}$

(e) (3 points) Estimate what is the first time $t \geq 0$ when the second object reverses its direction. No justification necessary.

at $t=5$ velocity will turn from positive to negative since S_2 turns from increasing to decreasing

4. (a) (5 points) Suppose $f(x)$ is the function defined below :

$$f(x) = \begin{cases} 2x+1, & \text{if } x \leq 2 \\ x^2 - 2x + b, & \text{if } x > 2 \end{cases}$$

For which values of the constant parameter b is $f(x)$ continuous everywhere ? Remember to justify your answer.

The only problem could be at $x_0 = 2$

we want $\lim_{x \to 2} f(x) = f(2)$

$f(2) = 5$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 2x+1 = 5$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 - 2x + b = b$

so we need $\boxed{b = 5}$

(b) (5 points) Suppose $f(x)$ is the function defined below :

$$f(x) = \begin{cases} 2x+1, & \text{if } x \leq 2 \\ x^2 - 2x + 5, & \text{if } x > 2 \end{cases}$$

According to the definition of derivative, $f'(2) = \lim_{h \to \boxed{0}} \frac{f(\boxed{2+h}) - f(\boxed{2})}{h}$

Fill in the blanks and carefully compute this limit showing all your steps.

$\lim_{h \to 0^-} \frac{f(2+h) - 5}{h} = \lim_{h \to 0^-} \frac{2(2+h)+1-5}{h} = \frac{2h}{h} = 2$

$\lim_{h \to 0^+} \frac{f(2+h) - 5}{h} = \lim_{h \to 0^+} \frac{(2+h)^2 - 2(2+h) + 5 - 5}{h} = \frac{4 + 4h + h^2 - 4 - 2h}{h}$

$\lim_{h \to 0^+} \frac{2h + h^2}{h} = 2 + h = 2$

so $\boxed{f'(2) = 2}$

5.

(a) (5 points) Let $g(x) = \sqrt{x} \cdot e^x + x^e + \frac{2}{x} + 3^4$. Compute $g'(x)$. You do not need to simplify your final answer.

$$g'(x) = \frac{1}{2\sqrt{x}} \cdot e^x + \sqrt{x}\, e^x + e x^{e-1} - \frac{2}{x^2}$$

(b) (5 points) Find the equation of the line tangent to the curve $y = x^2 + 1$ at the point $P = (1, -\frac{1}{4})$

Since $-\frac{1}{4} \neq 1^2 + 1$ P is not on the curve

Let $Q(x, x^2+1)$ be the point of tangency and m the slope of the tangent, then $m = \underbrace{2x}_{g'(x)} = \underbrace{\frac{x^2+1+\frac{1}{4}}{x-1}}_{\frac{\Delta y}{\Delta x}}$ so

$2x^2 - 2x = x^2 + \frac{5}{4}$ $\quad x^2 - 2x - \frac{5}{4} = 0$ $\quad x = \frac{2 \pm \sqrt{4+5}}{2} = \begin{cases} 5/2 \\ -1/2 \end{cases}$

So we have $Q_1(\frac{5}{2}, \frac{29}{4})$ and $Q_2(-\frac{1}{2}, \frac{5}{4})$ and 2 tangent lines

$$\boxed{y = \frac{29}{4} + 5\left(x - \frac{5}{2}\right)}$$

$$\boxed{y = \frac{5}{4} - \left(x + \frac{1}{2}\right)}$$