Math 124 I (Pezzoli)
Fall 2015
Midterm #1

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Name _____

TA: _____

Section: _____

Instructions:

- Your exam contains 5 problems.
- Your exam should contain 6 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides).

Problem #1 (20 pts) _____

Problem #2 (15 pts) _____

Problem #3 (18 pts) _____

Problem #4 (15 pts) _____

Problem #5 (12 pts) _____

TOTAL (80 pts) _____

1. Calculate the following limits. If the limit does not exist enter DNE. Make sure to show all steps.

(a) (5 points) $\lim_{x\to+\infty} \frac{3x^{10}+x^8+3}{x^{10}-3x^6+2} = \lim_{x\to+\infty} \frac{x^{10}\left(3+\frac{1}{x^2}+\frac{3}{x^{10}}\right)}{x^{10}\left(1-\frac{3}{x^4}+\frac{2}{x^{10}}\right)}$

$= \lim_{x\to+\infty} \frac{3+\frac{1}{x^2}+\frac{3}{x^{10}}}{1-\frac{3}{x^4}+\frac{2}{x^{10}}} = 3$

(b) (5 points) $\lim_{x\to1} \frac{e^x+7}{(x-1)^3} =$ DNE because

$\lim_{x\to1^+} \frac{e^x+7 \to e+7}{(x-1)^3 \to 0^+} = +\infty$

$\lim_{x\to1^-} \frac{e^x+7 \to e+7}{(x-1)^3 \to 0^-} = -\infty$

(c) (5 points) $\lim_{x\to5} \frac{(x^2-25)\cdot\sin x}{(x-5)\cdot\cos x} = (x+5)\frac{\sin x}{\cos x} = 10\cdot\frac{\sin 5}{\cos 5}$

(c) (5 points) $\lim_{x\to0} \frac{x}{\sqrt{x+4}-2} \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x\left(\sqrt{x+4}+2\right)}{x+4-4} =$

$= \lim_{x\to0} \sqrt{x+4}+2 = 4$

2. Given the function

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) + 3, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$

(a) (5 points) Calculate $\lim_{x \to 0} f(x)$. Make sure to show all steps.

$-1 \leq \sin(\frac{1}{x}) \leq 1$ therefore $-x^2 \leq x^2 \sin(\frac{1}{x}) \leq x^2$ and

by the squeeze th $\lim_{x \to 0} x^2 \sin(\frac{1}{x}) = 0$

so $\lim_{x \to 0} f(x) = \lim_{x \to 0} x^2 \sin(\frac{1}{x}) + 3 = 3$

(b) (5 points) Is f continuous at 0 ?

No because $\lim_{x \to 0} f(x) \neq f(0)$

(c) (5 points) Find all values of the parameter c, if any, such that the function

$$g(x) = \begin{cases} x^2 \sin(\frac{1}{x}) + 3, & \text{if } x \neq 0 \\ c, & \text{if } x = 0 \end{cases}$$

is differentiable at 0.

We need to look at $\lim_{h \to 0} \frac{g(0+h) - g(0)}{h} = \lim_{h \to 0} \frac{h^2 \sin(\frac{1}{h}) + 3 - c}{h}$

We can use the squeeze th again to argue that $\lim_{h \to 0} h \sin(\frac{1}{h}) = 0$ since from $-1 \leq \sin \frac{1}{h} \leq 1$ we get

$-h \leq h \sin \frac{1}{h} \leq h$ if $h \geq 0$ so $\lim_{h \to 0^+} h \sin \frac{1}{h} = 0$ and

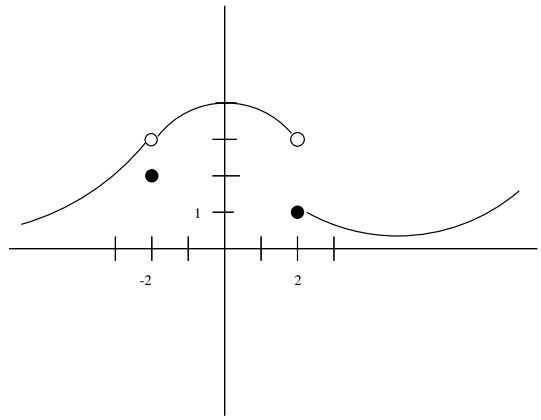
$-h \geq h \sin \frac{1}{h} \geq h$ if $h < 0$ so $\lim_{h \to 0^-} h \sin \frac{1}{h} = 0$

so if $c = 3$ $\lim_{h \to 0} \frac{g(0+h) - g(0)}{h} = 0$ and g is differentiable

if $c \neq 3$ by part a) g is not continuous at 0

so it cannot be differentiable, so $\boxed{c = 3}$ is the only value

3. Below is the graph of a function $y = f(x)$. Compute (no justification necessary): (If a limit does not exist enter DNE)



(a) (3 points) $\lim_{x\to 0} f(x) =$ 4

(b) (3 points) $\lim_{x\to 2^-} f(x) =$ 3

(c) (3 points) $\lim_{x\to 2} f(x) =$ DNE

(d) (3 points) $\lim_{x\to -2} f(x) =$ 3

(e) (3 points) $\lim_{x\to 0} \frac{f(x)}{x} =$ DNE

(f) (3 points) $\lim_{h\to 0} \frac{f(h)-4}{h} =$ $f'(0) = 0$

4. Calculate the derivatives of the following functions:

a) (5 points) $f(x) = x^{\sqrt{2}} + \sqrt{2} \cdot x^2 + \pi$

$\sqrt{2} \cdot x^{\sqrt{2}-1} + 2\sqrt{2} \cdot x$

b) (5 points) $g(x) = \dfrac{x \cdot e^x}{x+2}$

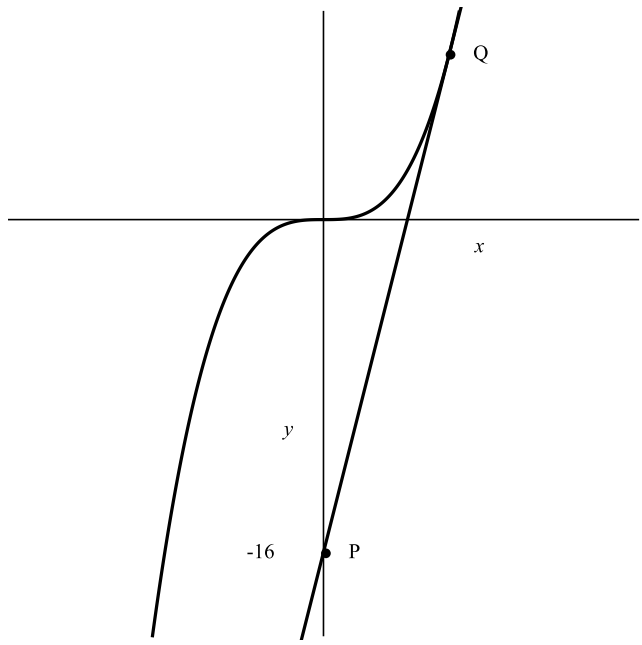
$$\frac{(e^x + xe^x)(x+2) - xe^x}{(x+2)^2} = \frac{2e^x + x^2e^x + 2xe^x}{(x+2)^2} =$$

$$e^x \frac{(x^2+2x+2)}{(x+2)^2}$$

c) (5 points) $f(x) = \frac{\sqrt{x}}{e^x}$

$$\frac{\frac{1}{2\sqrt{x}} \cdot e^x - e^x \cdot \sqrt{x}}{(e^x)^2} = \frac{\frac{1}{2\sqrt{x}} - \sqrt{x}}{e^x} = \frac{1-2x}{2\sqrt{x}\, e^{\cancel{x}}}$$

5. (15 points) The graph below shows the graph of $f(x) = x^3$ and a line L going through the point $P = (0, -16)$ and some point Q on the curve.



(a) (6 points) Find the equation of the tangent line to the graph of f(x) going through the point (-2,-8).

$f'(x) = 3x^2 \qquad f'(-2) = 3 \cdot 4 = 12$

$y = -8 + 12(x+2)$

(b) (6 points) The line L is tangent to the curve at Q. Find the exact equation of the line (an estimate based on the graph will receive no credit).

$Q = (x, y)$

$m = 3x^2 = \frac{y+16}{x} = \frac{x^3+16}{x}$, so $3x^2 = \frac{x^3+16}{x}$

$3x^3 = x^3 + 16 \qquad x^3 = 8 \qquad x = 2 \qquad f'(2) = 12$

$y = 2^3 + 12(x-2)$