

Name _____

Math 124A/B First Midterm — 100 points 8:30–9:50 a.m. Oct. 27, 2015

Please read each problem carefully, and show all your work clearly. You may have one sheet ($8\frac{1}{2} \times 11$ inches, both sides) of notes in your own writing, and you may use a regulation-model calculator. Non-native speakers of English may have a physical-copy dictionary. You may use derivative formulas in Problem 1, but not in any other problem.

1. (15 points) Find the derivative of each function (a)-(c). You may use derivative formulas, but **before you do that simplify the function, if possible, using algebra.** (But your answer does not have to be simplified.)

(a) $\frac{x^2}{e^{-x}\sqrt[3]{x}}$

(b) $\frac{(1 + \sqrt{x})(1 + x)(1 - \sqrt{x})}{\sec(x)}$

(c) $\frac{1 + x - 3x^2}{1 + x^3}$

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2. (20 points) Find the following limits. Please show your work clearly.

$$\begin{aligned} \text{(a)} \quad & \lim_{h \rightarrow 0} \frac{\frac{1}{t+h} - \frac{1}{t}}{h} & \text{(b)} \quad & \lim_{x \rightarrow -2^-} \frac{|x+2|}{x^2 + 7x + 10} & \text{(c)} \quad & \lim_{x \rightarrow 0^+} \frac{3^{2+(1/x)} + 5x^{-2} + 7x^2}{3^{5+1/x} + 6x^{-2} + 3x^2} \\ \text{(d)} \quad & \lim_{x \rightarrow 0^-} \frac{3^{2+(1/x)} + 5x^{-2} + 7x^2}{3^{5+1/x} + 6x^{-2} + 3x^2} & \text{(e)} \quad & \lim_{x \rightarrow +\infty} \frac{3^{2+(1/x)} + 5x^{-2} + 7x^2}{3^{5+1/x} + 6x^{-2} + 3x^2} \end{aligned}$$

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3. (20 points) Use limits to find a formula for the slope of the tangent line to the graph of $y = f(x)$ at the point $(x, f(x))$ if $f(x) = \sqrt{a + bx}$ (where a and b are constants). Please show your work clearly.

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4. (20 points) Sketch a graph of $y = f(x) = \cot\left(\frac{\pi}{3}x\right)$ for $0 < x < 3$, using the same scale for the x - and y -axes. Label the points on the curve where $x = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$. (Give their y -coordinates in exact form, that is, in terms of squareroots, **not** as a decimal.) Then sketch the graph of the derivative function $f'(x)$. Use graphical differentiation to estimate the derivative values.

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5. (25 points) In this problem use exact numbers (involving squareroots, π , etc.) rather than decimals; in other words, do not use a calculator for this problem. An object travels counterclockwise at constant angular velocity of 1 revolution every 8 seconds around the circle having equation $x^2 + y^2 = 200$. At time $t = 0$ it is at the right-most point on the circle (which is on the x -axis). After 11 seconds it flies off the circle, traveling at constant velocity along the tangent line. (You might find it useful to draw a picture showing the circle and the tangent line.) First find the radius and circumference of the circle and the speed of the object (which has constant speed, although the direction of its velocity vector changes). Also find the coordinates of the point where it flies off the circle. Then find parametric equations for the object's motion both before and after it flies off the circle, that is, both for $0 \leq t \leq 11$ and for $t \geq 11$. Please show your work clearly and neatly.