Math 124 Fall 2020 Final Exam

- **This exam starts at 1:30 pm and ends at 3:30 pm Seattle time.** You have until 3:45 pm to upload your solutions to gradescope. Set an alarm for yourself for 3:30 pm so you get your exam into gradescope ON TIME.
- Write the solutions to the problems on white or lined paper. When you are done upload your pages to gradecope and mark each question. Start each question on a new page to make it easier to mark your pages.
- In order to receive credit, **you must show all of your work**. If you do not indicate the way in which you solved a problem, you may get no credit for it, even if your answer is correct.
- You can use a two sided page of notes and a scientific calculator. You may not use any other sources online or offline. Any answer without supporting work will NOT get credit.
- Unless otherwise indicated, each question requires exact answers. Decimal approximations may not get full credit.
- This exam has 6 questions. Questions start on the next page.

GOOD LUCK!

Questions

1. Answer the following.

   (a) (6 points) Let $f(x) = \sqrt{x^2 - 2}$. Calculate $f'(x)$ using the limit definition of the derivative.

   (b) (6 points) Evaluate the limit
   $$\lim_{x \to \infty} \frac{x \arctan(x)}{2x + 1}.$$

   (c) (6 points) Evaluate the limit
   $$\lim_{x \to 1^+} \left(\frac{x}{x - 1} - \frac{1}{\ln x} \right).$$

   (d) (8 points) Evaluate the limit
   $$\lim_{x \to 0} \left[\ln\left(\cos(x)\right)\right]^x.$$

2. (12 points) Consider the curve defined by the equation
   $$y - 1 = \arcsin(2xy).$$

   (a) Find an expression for $\dfrac{dy}{dx}$.

   (b) Show that there are no points on the curve where the tangent line is horizontal.

   (c) Use the linear approximation at $(0, 1)$ to estimate the y value of the point $(-0.1, y)$.

3. (14 points) A figure-8 curve is given by
   $$x = \sin t \qquad \text{and} \qquad y = \sin(2t)$$
   where $0 \le t \le 2\pi$.

   (a) Find the points on the curve where the tangent lines are horizontal or vertical.
   (b) Find all the inflection points on the curve.

4. (16 points) An 8 cubic centimeter lump of putty is divided into two parts. One is shaped into a regular tetrahedron and the other into a cube.
A regular tetrahedron, also known as a triangular pyramid, is composed of four equilateral triangular faces, six straight edges, and four vertex corners. If the length of one of its edges is b, its volume and surface area are given by

$$\text{Volume} = \frac{b^3}{6\sqrt{2}} \qquad\qquad \text{Surface Area} = \sqrt{3}b^2.$$

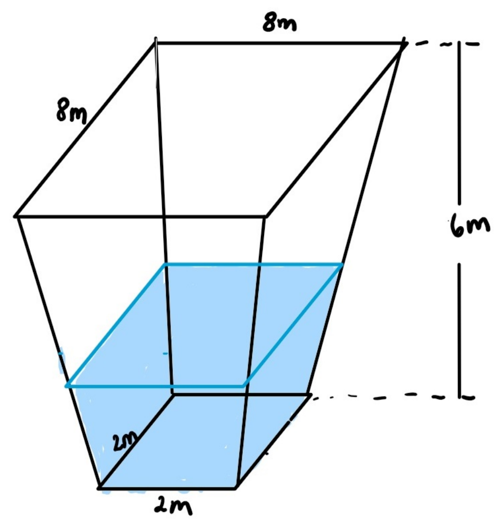
Find the dimensions of the tetrahedron and the cube that will maximize the total surface area of the two shapes. Make sure you justify that the answer you found is really a maximum.

5. (16 points) A new pond has the form of an inverted, capped square pyramid as shown on the right. The smaller square at the bottom has side length 2m, the top square has side length 8m. Water is filled in at a rate of $\frac{19}{3}$ cubic meters per hour. At what rate is the water level rising exactly 1 hour after the water started to fill the pond?

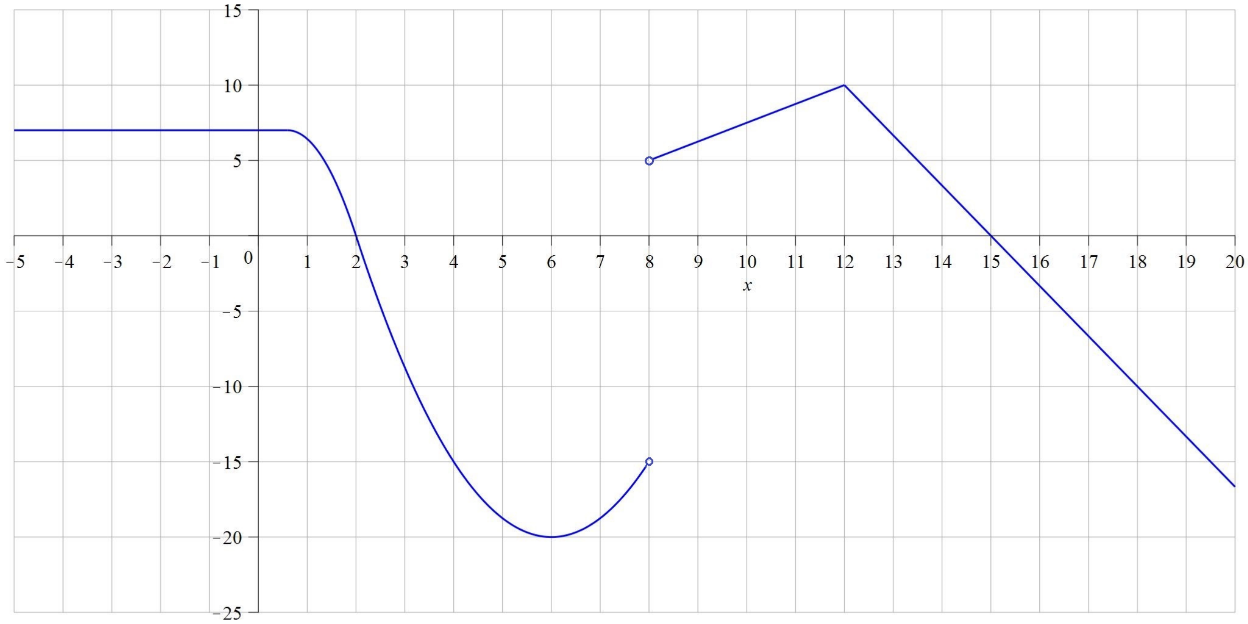
Hint: The volume of a full square pyramid is

$$V = \frac{1}{3}a^2h,$$

where a is the side length of the pyramid's base and h is the height of the pyramid.



6. (16 points) The function $f(x)$ is continuous on its domain $[-5, 20]$. The following is the graph of the **derivative** of the function $f(x)$. Use the graph to answer the questions below. In some cases, you may not be able to read an exact value from the graph. If this happens, just approximate the value as well as you can.



The graph of the derivative $y = \frac{d}{dx} f(x)$.

(a) List all the intervals where the function $f(x)$ is increasing.

(b) List all the intervals where the graph of the function $f(x)$ is concave up.

(c) List the x coordinates of all critical points of $f(x)$ in the interval $(-5, 20)$ and identify them as local minimum, local maximum, or neither.

(d) What is the $\lim_{h \to 0} \frac{f'(6+h) - f'(6)}{h}$?

(e) What is the $\lim_{h \to 0^+} \frac{f(8+h) - f(8)}{h}$?

(f) Given $f(14) = 0$, what is the value of $\lim_{x \to 14} \frac{f(x)}{x - 14}$?