Math 124 **Final Examination** **Spring 2019**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	12	
4	12	

Question	Points	Score
5	13	
6	12	
7	12	
8	15	
Total	100	

1. (12 total points) Calculate the derivatives of the following functions. You do not need to simplify your answers.

   (a) (4 points) $g(x) = \dfrac{\sin(x^2)}{1 + e^{x^3}}$

   (b) (4 points) $f(t) = \ln(2t)\sqrt{e^{\sin t} + 1}$

   (c) (4 points) $y = (2x + 1)^{\sqrt[3]{x}}$

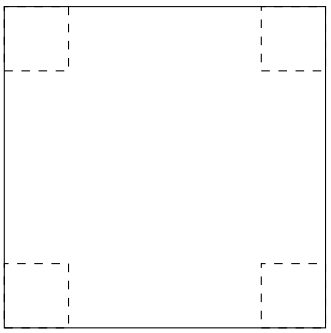
2. (12 total points) Evaluate the following limits. Show the algebra work where applicable. If the limit does not exist, explain why not.

   (a) (4 points) $\lim_{x\to 1} \dfrac{x^3 - x}{x^3 - 2x^2 - 3x}$

   (b) (4 points) $\lim_{x\to\infty} \left(\sqrt{16x^2 + 4x} - 4x\right)$

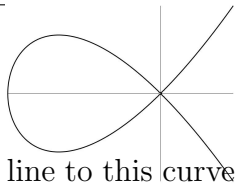
   (c) (4 points) $\lim_{x\to\infty} e^{-\frac{\sin(1/x)}{x}}$

3. (12 points) From a square piece of cardboard paper of area size 9 m^2, squares of the same size are cut off from each corner of the paper. This allows for folding the paper to a box without a top. What values for the lengths of the squares only make sense? How do we have to choose the length of the squares to maximize the volume of the box?
Verify that your answer is indeed a maximum.



4. (12 points) A balloon is being filled with air. After t seconds, air is being pumped into the balloon at a rate of $2t + 10$ cm^3/sec. Suppose that after 10 seconds, the volume of the balloon is 100 cm^3. What is the rate that the surface area of the balloon is changing at $t = 10$ seconds?

   (Recall the formulae for the volume and surface area of a sphere: $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.)



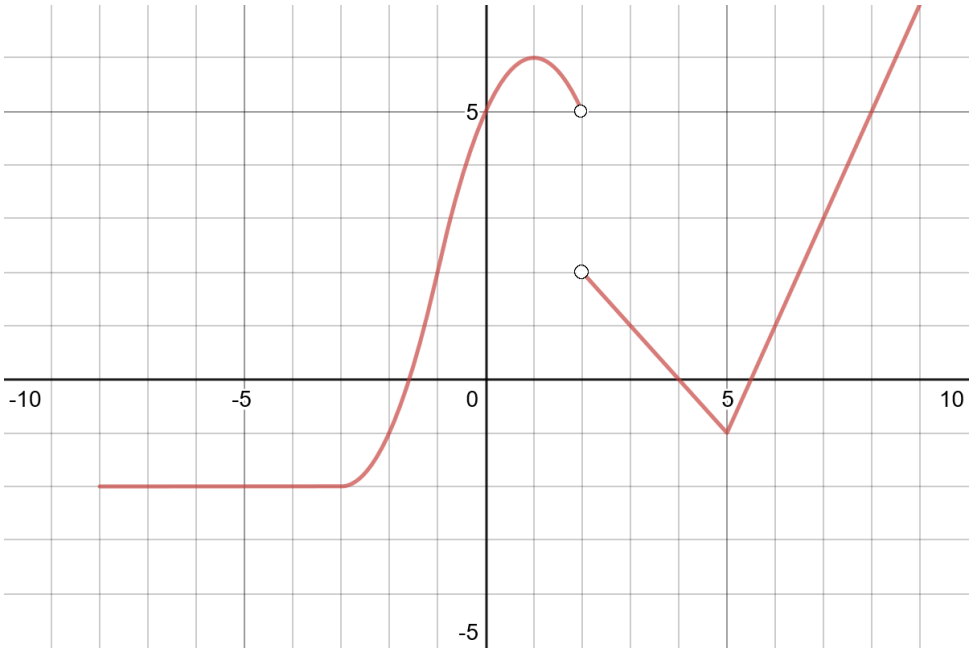
5. (13 total points) Consider the curve with equation $y^2 = x^3 + 3x^2$.

   (a) (5 points) With implicit differentiation, find the equation of the tangent line to this curve at the point $(1, -2)$.

   (b) (5 points) At which points (a, b) does the curve have a horizontal tangent?

   (c) (3 points) Are there any vertical tangents? Explain your answer.

6. (12 total points) The following graph is the graph of the **derivative** of a function f. The domain of f is the interval $[-9, 10]$. Answer the following questions:



Graph of f'

(a) (2 points) List all intervals where f is increasing.

(b) (2 points) At which values is f differentiable?

(c) (2 points) List the critical numbers of f and identify them as a local minimum, maximum or neither.

(d) (2 points) List all intervals on which the graph of $y = f(x)$ is concave down.

(e) (2 points) Determine $\lim_{h \to 0} \dfrac{f'(4+h) - f'(4)}{h}$

(f) (2 points) If $f(-2) = -1$, use the tangent line approximation to estimate $f(-2.1)$

7. (12 total points) An ant crawls according to the equations

$$x(t) = e^{t^2-t}\cos(\pi t^3) \qquad \text{and} \qquad y(t) = e^{t^2-t}\sin(\pi t^2)$$

beginning at $t = 0$.

(a) (4 points) At what times t in the interval $[\,0, 2\,]$ does the ant cross the x-axis?

(b) (4 points) What is the ant's speed at $t = 1$?

(c) (4 points) What is the equation of the line tangent to the ant's path at $t = 1$?

8. (15 total points) Consider the function $f(x) = \dfrac{x^3 - 1}{x^3 + 1}$.

   (a) (3 points) Find all horizontal and vertical asymptotes of $y = f(x)$.

   (b) (3 points) Determine the critical numbers of $f(x)$.

   (c) (3 points) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

Continued on the next page.

Recall that the function is $f(x) = \dfrac{x^3 - 1}{x^3 + 1}$.

(d) (3 points) Determine the inflection points of $f(x)$.

(e) (3 points) Sketch the graph of $y = f(x)$ on the axis provided below. Be sure to include asymptotes in your picture as well as to mark the coordinates of critical and inflection points.