Math 124 **Final Examination** **Autumn 2019**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 11 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	8	
2	8	
3	8	
4	10	
5	12	

Question	Points	Score
6	10	
7	12	
8	10	
9	10	
10	12	
Total	100	

1. (8 total points) Calculate the derivatives of the following functions. You do not need to simplify your answers.

   (a) (4 points) $g(x) = 2\ln x + x\sqrt{5x^3 + \sin\left(e^{7x}\right)}$

   (b) (4 points) $f(x) = 10^x + \dfrac{2\cos x}{x^2+1}$

2. (8 total points) Solve the following problems. Recall that $\tan^{-1}(x)$ and $\arctan(x)$ are the same function.

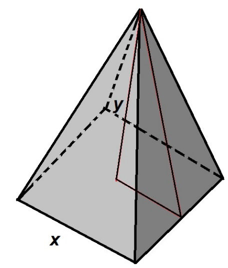
   (a) (4 points) Let $y = (1+x^2)^{\tan^{-1}(x)}$. Compute $\dfrac{dy}{dx}$.

   (b) (4 points) Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Calculate $f'(0)$, using the definition of derivative.

3. (8 total points) Evaluate the following limits. Show the algebra work where applicable. If the limit does not exist, explain why not.

   (a) (4 points) $\lim_{x \to 0} \frac{2 - \sqrt{x^2 + 4}}{\sin^2 x}$

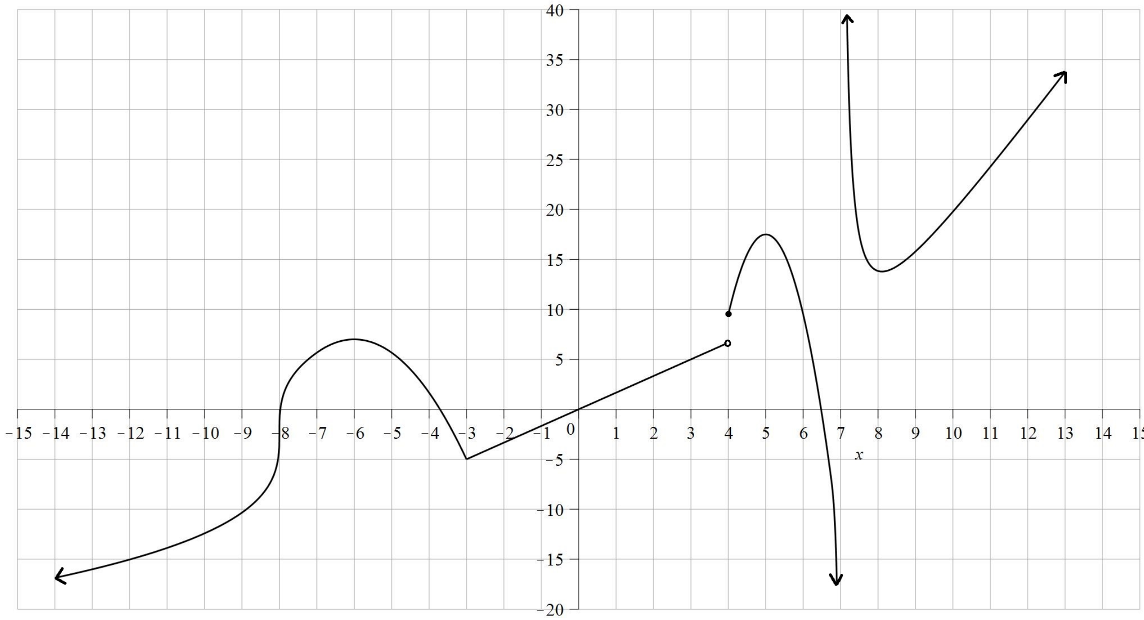
   (b) (4 points) $\lim_{x \to \infty} \frac{x + 1}{\sqrt{2x^2 + x + 1}}$



4. (10 points) Find the height y and length of a side of its base x of a square pyramid with congruent isosceles triangle sides whose volume is $\frac{1}{6}$ cubic meters and whose lateral surface area is as small as possible.
(The lateral surface area includes the sides but excludes the base of the pyramid.)

   Justify that the dimensions you found give minimum lateral surface area.

5. (12 total points) The following is the graph of $y = f(x)$ with domain all real numbers $x \neq 7$. For limit questions your answer must be one of DNE, ∞, $-\infty$ or a number.



(a) (2 points) $\lim_{x \to 4} f(x) =$

(b) (2 points) $\lim_{x \to 7^+} f(x) =$

(c) (2 points) List all the intervals where the derivative $f'(x)$ is negative.

(d) (2 points) List all critical values of $f(x)$.

(e) (2 points) $\lim_{x \to 0} \dfrac{f(x)}{x} =$

(f) (2 points) $\lim_{h \to 0} \dfrac{f(3+h) - 5}{h} =$

6. (10 points) Find the equations of the tangents to the curve given parametrically by

$$x = 3t^2 + 2,\ y = 2t^3 + 3$$

that pass through the point $(5, 5)$.

7. (12 total points) Consider the curve defined by the equation

$$x^3 + y^4 - 6x^2 - 2x = 0.$$

In this problem, round all your numerical answers to 3 decimal places.

(a) (4 points) Find an expression for $\frac{dy}{dx}$.

(b) (4 points) Find the coordinates of all points on the curve where the tangent line is vertical.

(c) (4 points) Find the coordinates of all points on the curve where the tangent line is horizontal.

8. (10 points) A spherical snowball melts so that its volume decreases at a rate of $\frac{1}{5}$ cm^3/min. How fast is its surface area decreasing when its radius is 5 cm?
(The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$.)

9. (10 points) Use linear approximation near $(0, 0)$ to estimate the y value of a point $(0.004, y)$ on the curve $3y = \tan^{-1}(y - x)$.

10. (12 total points) Consider the function $f(x) = x - 2\sin(x)$ on the domain $D = [-\pi, 3\pi]$.

    (a) (3 points) Find the intervals in D on which $y = f(x)$ is increasing, and the intervals on which f is decreasing.

    (b) (3 points) Find the intervals in D on which $y = f(x)$ is concave up and concave down.

Continued on the next page.

Recall that the function is $f(x) = x - 2\sin(x)$ *on the domain* $D = [-\pi, 3\pi]$.

(c) (3 points) What are the local minima and maxima? What are the global minimum and maximum?

(d) (3 points) Use your results in parts (a)-(c) to graph $y = f(x)$. Include both coordinates of all local minima and maxima, inflection points, x-intercepts, and y-intercepts.