Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	15	
2	15	
3	13	
4	13	

Question	Points	Score
5	12	
6	14	
7	18	
Total	100	

1. (15 total points) Calculate the derivatives of the following functions. You do not need to simplify your answers.

   (a) (5 points) $f(x) = \ln\left[\tan^2 x + \ln\left(2 + \sin^2 x\right)\right]$

   (b) (5 points) $g(x) = x \cdot \cos\left(\dfrac{7x^3 + 5}{x^2}\right)$.

   (c) (5 points) $h(t) = (t+1)^{\sqrt{t}}$

2. (15 total points) Evaluate the following limits. Your answer should be a number, ∞, $-\infty$ or DNE.

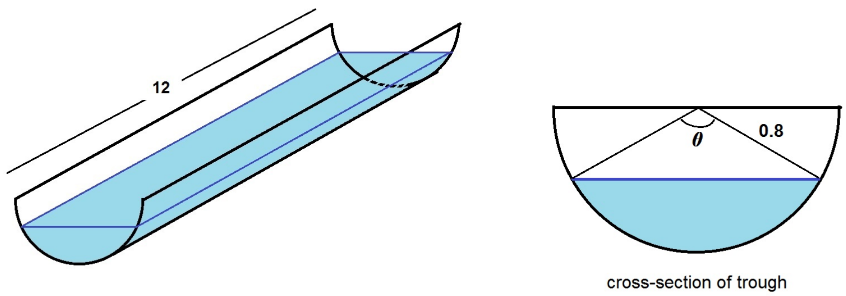
   (a) (5 points) $\displaystyle\lim_{x\to 0^+} \frac{1}{\sqrt{x}} - \frac{1}{\sin x}$

   (b) (5 points) $\displaystyle\lim_{x\to\infty} 3x - \sqrt{9x^2+5x}$

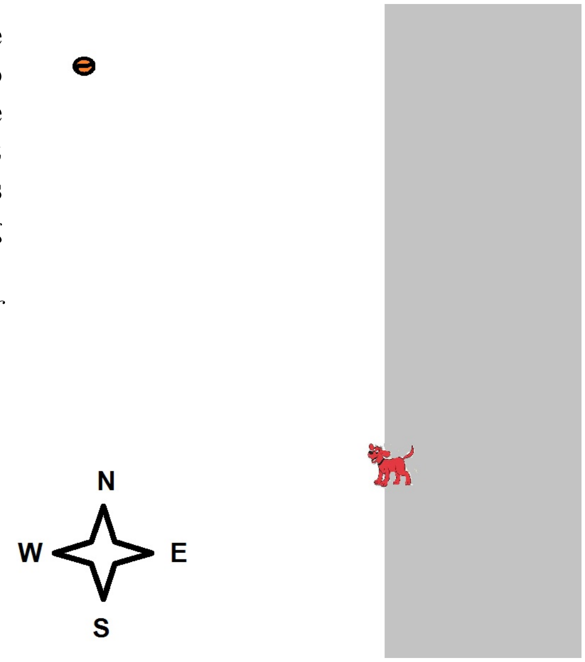
   (c) (5 points) $\displaystyle\lim_{x\to 2} \frac{x^2+x-6}{x^3-4x^2+4x}$

3. (13 points) A water trough is 12 meters long with a semi-circular cross section of radius 0.8 meters. The trough and its cross section are shown below. The pictures are not to scale. The trough is being filled at a rate of 0.3 cubic meters per minute. At what rate is the angle θ shown changing when it is $\frac{2\pi}{3}$ radians? Give an exact answer with units.

   (Hint: Start by calculating the shaded area in the cross section.)



4. (13 points) Suppose you are at the beach, standing at the edge of the water with your dog. As you look westward into the ocean, you throw a tennis ball out into the water. The ball lands in the water 20 meters North and 13 meters West of where you are standing. The dog swims at 5 m/s but runs at 13 m/s. How far North does the dog run before jumping in the water to minimize his time to get to the ball?
Assume the coastline is straight and ignore both water currents and wind drag.



5. (12 total points) Consider the curve defined by the equation $x^4 - 2x^2y^2 = -56$.

   (a) (6 points) Find all points on the curve with a horizontal tangent line.

   (b) (3 points) Find the equation of the tangent line at the point $(2, 3)$.

   (c) (3 points) Let P be the point on the curve near $(2, 3)$ with x-coordinate 2.1. Find an approximate value of the y-coordinate of P. Round your answer to three digits after the decimal.

6. (14 total points) Consider the curve given by the following parametric equations:

$$x(t) = t^3 - 4t, \quad y(t) = t^2; \qquad -\infty < t < \infty$$

(a) (7 points) Find the equations of the <u>two</u> tangent lines to the curve at the point $(0, 4)$.

(b) (7 points) Let $g(t)$ be the slope of the curve at time t. Compute the instantaneous rate of change of $g(t)$ at time $t = 1$.

7. (18 total points)Consider the function $f(x) = \dfrac{x^2}{x^2 - 4}$.

   (a) (3 points) Find the vertical asymptotes of $y = f(x)$, if there are any.

   (b) (3 points) Find the horizontal asymptotes of $y = f(x)$, if there are any.

   (c) (3 points) Find the intervals where the function is increasing and the intervals where the function is decreasing.

Recall that the function is $f(x) = \dfrac{x^2}{x^2 - 4}$.

(d) (3 points) Find the intervals where the function is concave up and the intervals where the function is concave down.

(e) (3 points) Find all critical numbers of the function and characterize them as the x-coordinates of local minima, local maxima, or neither.

(f) (3 points) Sketch the graph of $y = f(x)$ on the axis provides below. Be sure to include asymptotes in your picture. Also mark the coordinates of all local maxima, local minima, and inflection points (if any exist).