Math 124 **Final Examination** **Winter 2017**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can use only Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	12	
4	12	

Question	Points	Score
5	12	
6	12	
7	12	
8	16	
Total	100	

1. (12 total points) Find the derivative of the following functions.

   (a) (4 points) $f(x) = \dfrac{3x}{e^{\sin(\pi x)}}$

   (b) (4 points) $f(x) = x^{2^x}$

   (c) (4 points) $f(x) = \ln(kx) \cdot \sqrt{x^{2k}+3}$, where the constant $k > 0$.

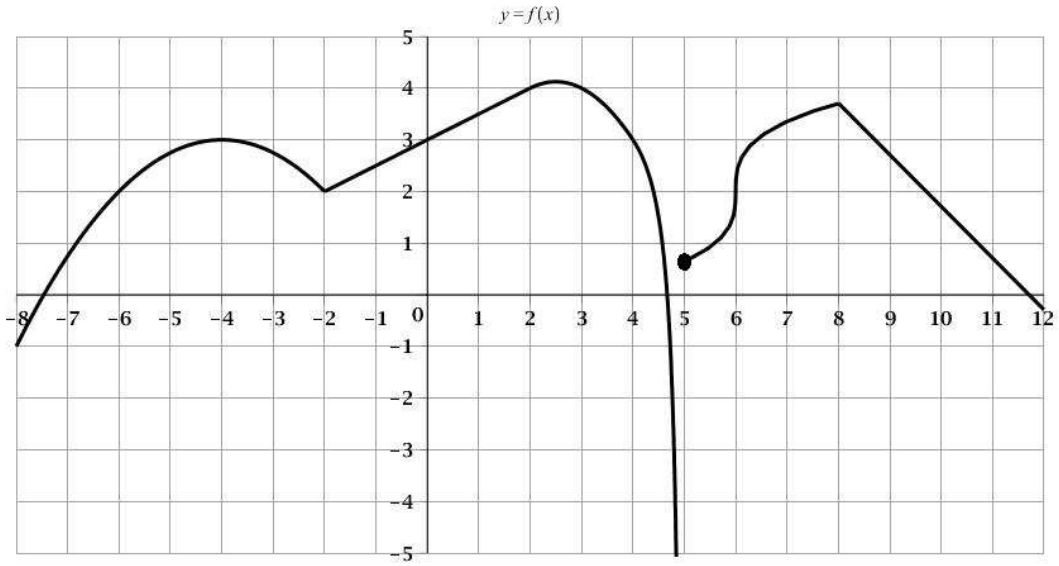
2. (12 total points) Find the following limits. In each case your answer should be either a number, $+\infty$, $-\infty$ or DNE. Please show your work.

   (a) (4 points) $\lim_{t \longrightarrow 2^-} \dfrac{t^2-4}{|t-2|}$

   (b) (4 points) $\lim_{x \longrightarrow \infty} \left(x - \sqrt{x^2-10x}\right)$

   (c) (4 points) $\lim_{x \longrightarrow \infty} \dfrac{2x^2+3x\ln x+2^{-x}}{5x^2+9x\ln x+\pi \cdot 2^{-x}}$

3. (12 total points) The following is the graph of the function $f(x)$ with domain $-8 \leq x \leq 12$. The vertical line $x = 5$ is an asymptote. Answer the following questions based on the graph. You do not need to justify your answers on this problem.



(a) (2 points) List all intervals where the derivative $f'(x)$ is increasing.

(b) (2 points) $f'(-1) =$

(c) (2 points) $f''(0) =$

(d) (2 points) $\lim_{x \to -2} f(x) =$

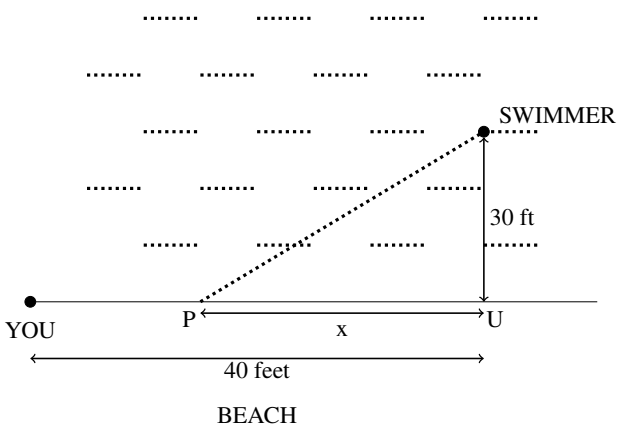
(e) (2 points) $\lim_{x \to -4} \dfrac{f(x) - 3}{x + 4} =$

(f) (2 points) $\lim_{h \to 0} \dfrac{f(6+h) - 2}{h} =$

4. (12 points) You are standing on the beach when you notice a swimmer in distress. He is 30 feet from shore. At the point on the shore nearest the swimmer there is a big beach umbrella (marked U in the picture below). You are 40 feet down the beach from the umbrella. You want to run along the beach to a point P, then jump into the water and swim in a straight line towards him. Your running speed is 10 feet/sec and your swimming speed is 5 feet/sec.

   What should the distance x between P and the umbrella U be if you want to minimize the time it takes you to reach the swimmer?

   Verify that your answer is a minumum.



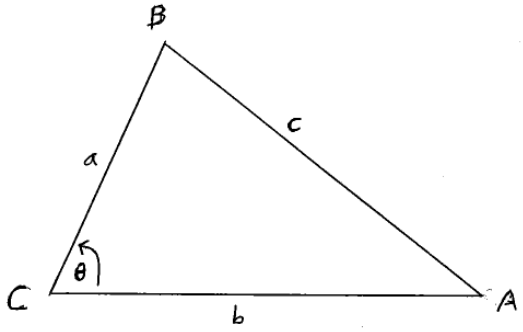
5. (12 total points) For time $t > 0$ seconds, a particle moves according to the parametric equations

$$x(t) = 3t^2 - t - 9, \quad y(t) = 9t^3 - 8\ln t$$

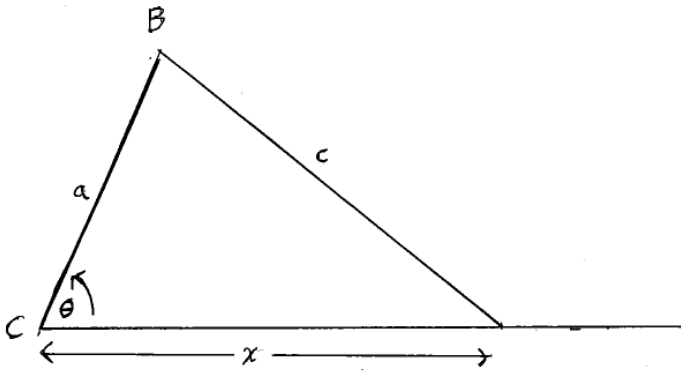
(a) (6 points) Find the time t at which the path of the particle in the xy-plane has a horizontal tangent.

(b) (6 points) At time $t = 1$ second, the particle departs from the parametric curve and continues along the tangent line to the curve at that point. The particle travels along this tangent line at a constant speed, preserving the horizontal and vertical velocities from the moment it left the parametric curve. How many seconds after leaving the curve will the particle cross the y-axis?

6. (12 total points) In this problem use the Law of Cosines (see figure to the right):
$c^2 = a^2 + b^2 - 2ab\cos(\theta)$.



In the second diagram to the right the sides a and c are metal bars. As you change θ, the lower end of the side c moves left or right along the horizontal line. (There are hinges at C and B so that the angles there can change.) In this problem take $a = 5$ meters and $c = 7$ meters.



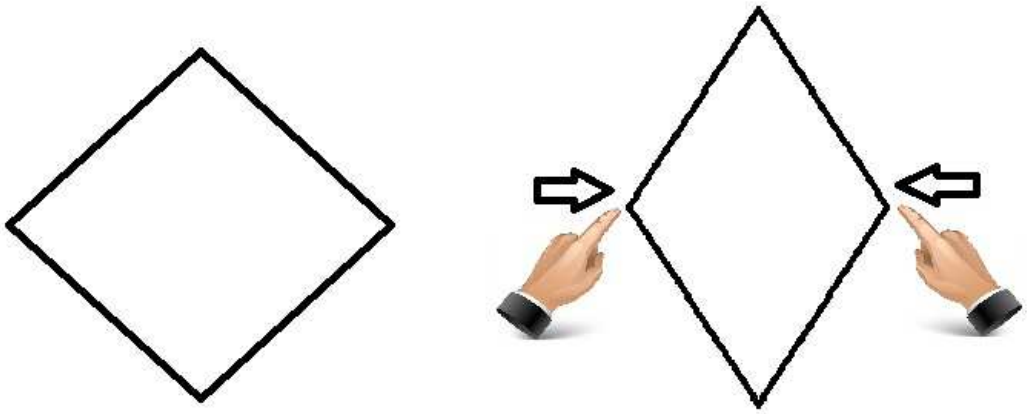
(a) (4 points) Find a formula for $\dfrac{dx}{d\theta}$ in terms of x and θ.

(b) (4 points) When $\theta = \frac{\pi}{3}$ radians find both x and $\dfrac{dx}{d\theta}$. Give units in your answer.

(c) (4 points) In the situation in part (b), suppose that θ is increased by 0.01 radians. What is the new value of x? Use linearization (the tangent line approximation).

7. (12 points) A square is formed from four 15 cm rods, connected by hinges that can move. You push the opposite corners so that the horizontal distance between them is decreasing at a rate of 0.6 cm/sec. (The distance between the finger tips in the picture is decreasing at this rate.)

   At what rate is the area enclosed by the rods changing when the horizontal distance between the opposite corners is 18 cm?



8. (16 total points) Let $f(x)$ be the function

$$f(x) = (2x+5)e^{(-x/2)}$$

(a) (2 points) Give the (x,y)-coordinates of all x-intercepts and y-intercepts of $y = f(x)$.

(b) (2 points) Find the following limits.

i. $\lim_{x\to\infty} f(x)$

ii. $\lim_{x\to-\infty} f(x)$

(c) (3 points) Find all intervals over which $f(x)$ is increasing.

8. (continued) Recall that the function is $f(x) = (2x+5)e^{(-x/2)}$

   (d) (3 points) Find all intervals over which $f(x)$ is concave down.

   (e) (6 points) Sketch the graph of $y = f(x)$ using the grid below. Clearly label the (x, y) coordinates of all critical points and all points of inflection.