Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK).
  Do not share notes. No photocopied materials are allowed.
- You can use only Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	16	
4	10	

Question	Points	Score
5	12	
6	12	
7	12	
8	14	
Total	100	

1. (12 total points) Find the derivative of the following functions. Do not simplify your answer.

   (a) (4 points) $f(x) = 10^x \cdot \tan^2 x$

   (b) (4 points) $g(x) = \dfrac{(e^x + 1)^5}{\sqrt{x^2 + 4}}$

   (c) (4 points) $y = (x^2 + 4)^{\ln x}$

2. (12 total points) Compute the following limits. If limits do not exist (including infinite limits), explain why.

   (a) (4 points) $\lim_{x \to 0} \sqrt{x} \, \cos\left(\frac{2}{x}\right)$

   (b) (4 points) $\lim_{x \to 2} \frac{x^{1/2} - 2^{1/2}}{x^{1/3} - 2^{1/3}}$

   (c) (4 points) $\lim_{x \to \infty} \left(x - \sqrt{x^2 + 3x}\,\right)$

3. (16 total points) Suppose the position of a moving particle is given by the parametric curve

$$x = \sin(t), \quad y = \sin(2t)$$

for $0 \leq t \leq 2\pi$. Answer the following questions.

(a) (3 points) How many times does the particle visit the origin?

(b) (4 points) Are there any points on the curve where both the vertical and the horizontal acceleration is zero? If so, what are they?

Continued on the next page...

3. (continued)

   (c) (4 points) Are there any times when both horizontal and vertical velocities are decreasing? If so, what are they?

   (d) (5 points) Suppose at time $t = 2\pi/3$ the particle leaves the curve and travels along the tangent at that point at constant speed. How long will it take for the particle to hit the y-axis?

4. (10 total points) Consider the equation

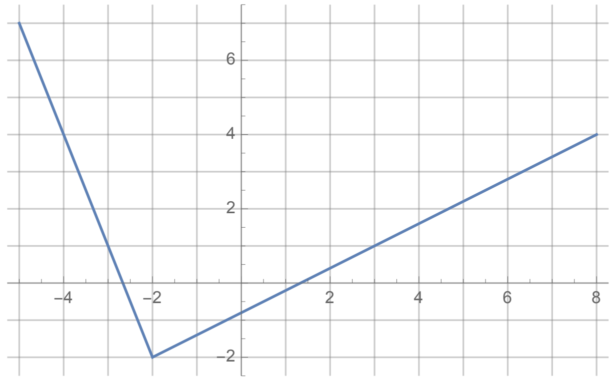
$$3(x^2 + y^2)^2 = 25(x^2 - y^2)$$

that describes the graph of a lemniscate.

(a) (5 points) Find dy/dx using implicit differentiation.

(b) (5 points) Find the equation of the tangent line to the graph of the lemniscate at the point $P(-2, 1)$.

5. (12 total points) Consider the graph of a function $f(x)$ given below. Answer the following questions.



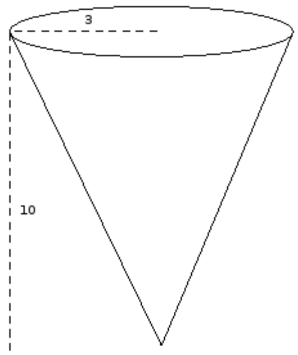
(a) (3 points) If $g(x) = f(f(x))$, what is $g'(-3)$?

(b) (3 points) For what values of x does $y = g(x)$ have a horizontal tangent? If there are no such points, explain why.

(c) (3 points) For what values of x is $[f(x)]^2$ decreasing?

(d) (3 points) Is $f'(x)$ continuous? If yes, say so. If not, what kind of discontinuity does it have?

6. (12 points) A paper cup has the shape of a cone with height 10 cm and radius 3 cm at the top. If water is poured into the cup at a rate of $2\text{cm}^3/\text{s}$, how fast is the water level rising when the water is 5 cm deep?



7. (12 points) A piece of wire 12 meters long is cut into two pieces. One piece is bent into a circle and the other is bent into an equilateral trangle. How should the wire be cut so that the total area enclosed is (i) a maximum? (ii) a minimum?

8. (14 total points) Let $f(x)$ be the function

$$y = f(x) = \frac{4}{x^2 - 4}$$

(a) (4 points) Find all intervals over which $f(x)$ is increasing or decreasing.

(b) (4 points) Find all intervals over which $f(x)$ is concave up or cancave down.

8. (continued) Recall the function $y = f(x) = \dfrac{4}{x^2 - 4}$

(c) (2 points) Find the horizontal and vertical asymptotes.

(d) (4 points) Sketch the graph of $y = f(x)$ below. Clearly label the (x, y) coordinates of all critical points and all points of inflection.