Math 124 **Final Examination** **Autumn 2016**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form, for example $\frac{\pi}{3}$ or $5\sqrt{3}$.
- You can use only Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	12	
4	12	

Question	Points	Score
5	12	
6	12	
7	12	
8	16	
Total	100	

1. (12 total points) Find the derivative of the following functions.

   (a) (4 points) $g(x) = e^{-x^2} \arctan x$

   (b) (4 points) Suppose that $f(0) = \pi/4$ and $f'(0) = 3$. Let $h(x) = \ln(\tan(f(x)))$. Compute $h'(0)$.

   (c) (4 points) $y = (3 + 2\sin x)^{3x}$

2. (12 total points) Compute the following limits. If you apply L'Hôpital's rule then you must show that you have checked the hypotheses.

   (a) (4 points) $\lim_{x\to\infty}\left(\sqrt{x^4+7x^2}-x^2\right)$

   (b) (4 points) $\lim_{t\to 1}\dfrac{1-t+\ln t}{1+\cos(\pi t)}$

   (c) (4 points) $\lim_{x\to\infty}\dfrac{x}{2x-\sin x}$

3. (12 total points) An object is moving along an ellipse. Its location is given by the parametric equations

$$x(t) = 1 + 2\cos t \qquad y(t) = 2 + 4\sin t$$

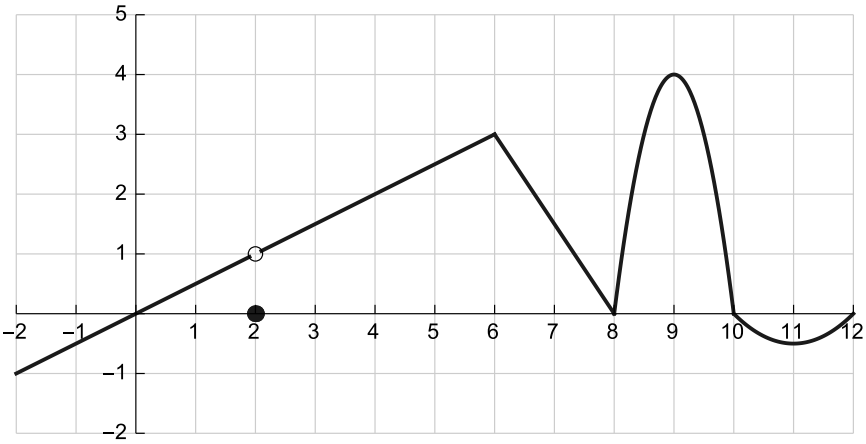
In this problem we take $0 \le t \le 2\pi$.

(a) (3 points) Find a formula that gives the slope of the tangent line to the path at time t as a function of t.

(b) (4 points) Find the equation of the tangent line at $t = \frac{\pi}{3}$.

(c) (5 points) Find all the values of t when the tangent line is perpendicular to the line $x - 2y = 3$.

4. (12 total points) For this problem, refer to the pictured graph of the function $y = f(x)$ on the interval [-2,12].



(a) (2 points) $\lim_{x\to 7} \dfrac{f(x)-f(7)}{x-7} =$

(b) (2 points) $\lim_{x\to 2} f'(x) =$

(c) (2 points) $\lim_{x\to 2} f''(x) =$

(d) (2 points) $\lim_{x\to 2} \dfrac{f(x)}{x} =$

(e) (2 points) Circle the smallest number in this list:

$$f'(0) \qquad f'(1) \qquad f'(7) \qquad f'(9) \qquad f'(11)$$

(f) (2 points) Give an interval (a,b) on which $f'(x)$ is increasing.

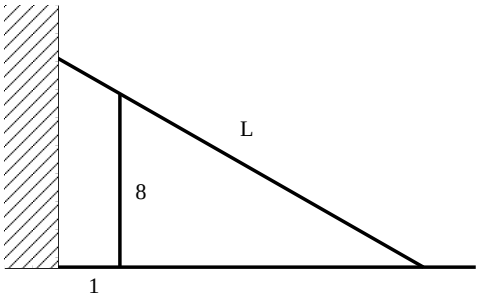
5. (12 total points) Consider the plane curve

$$2x^4 - 4xy + y^2 = 16.$$

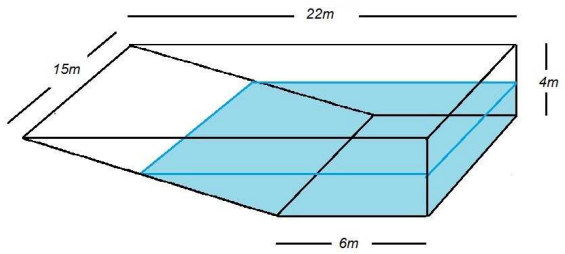
(a) (6 points) Use the tangent line approximation at $(0,4)$ to estimate the value of y when $x = -0.04$.

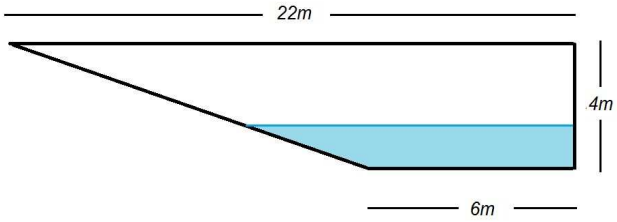
(b) (6 points) Find the second derivative $\frac{d^2y}{dx^2}$ at $(0,4)$ and use this to decide if the tangent line approximation is an overestimate or an underestimate near $(0,4)$. Explain your reasoning.

6. (12 points) The side wall of a building is to be braced by a beam which must pass over a parallel wall 8 feet high and 1 foot from the building. Find the length L of the shortest beam that can be used. Verify that your answer is a minumum.



7. (12 points) A pool is 15 meters wide and 22 meters long. The cross section is in the shape of a right trapezoid, one of the parallel sides being the length of the pool, and the other one equal to 6 meters. At its deepest point the pool is 4 meters deep. The pool and its cross section are pictured below. Water is pumped into the pool at a rate of 7 cubic meters per minute. How fast is the water level rising when the water level is 3 meters measured at its deepest end?





8. (16 total points) Let $f(x)$ be the function

$$y = f(x) = \frac{12}{x} - \frac{12}{x^2} + \frac{4}{x^3}$$

on the domain of all non-zero real numbers.

(a) (4 points) Find all intervals over which $f(x)$ is decreasing.

(b) (4 points) Find all intervals over which $f(x)$ is concave down.

8. (continued) Recall the function $y = f(x) = \dfrac{12}{x} - \dfrac{12}{x^2} + \dfrac{4}{x^3}$

(c) (4 points) Calculate the following limits.

1. $\lim_{x\to\infty} f(x)$

2. $\lim_{x\to-\infty} f(x)$

3. $\lim_{x\to0^+} f(x)$

4. $\lim_{x\to0^-} f(x)$

(d) (4 points) Sketch the graph $f(x)$ using the grid below. Clearly label the (x,y) coordinates of all critical points and all points of inflection.