Math 124 **Final Examination** **Autumn 2025**

Your Name

Your Signature

Student ID #

Quiz Section

December 6, 2025

Put a check next to your professor's name

☐ Prof. Jayadev Athreya ☐ Prof. Ebru Bekyel

☐ Prof. Fanny Dos Reis ☐ Prof. Guillermo Sanmarco

READ THE INSTRUCTIONS!

- *These exams will be scanned. Write your name and student number clearly. Do NOT write too close to the edges.*
- Turn off and stow away all cell phones, smart watches, and other similar devices. No ear buds, headphones, or any kind of connected devices allowed during the exam.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- **Give your answers in exact form.** For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- This exam has 11 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

Problem	Score	Problem	Score	Problem	Score
1 (12 pts)		4 (10 pts)		7 (11 pts)	
2 (15 pts)		5 (12 pts)		8 (19 pts)	
3 (9 pts)		6 (12 pts)		**Total**	

1. (12 total points) Find the following limits or explain why the limit does not exist. Give exact answers.

   (a) (4 points) $\lim_{x\to\infty} x\ln(x+2) - x\ln(x)$

   (b) (4 points) $\lim_{x\to 0^+} x^x$

   (c) (4 points) $\lim_{x\to\frac{\pi}{2}} \dfrac{\sin(3x)+1+3\cos(x)}{\cos(5x)+1-\sin(x)}$

2. (15 total points) Differentiate the following functions using the rules of differentiation. Do not simplify.

   (a) (5 points) $f(x) = e^{-\frac{x^2}{2}} + x^2 \cos(x)$

   (b) (5 points) $g(x) = \big(\sin(x^3)\big)^{\ln(x)}$

   (c) (5 points) $h(x) = \dfrac{\arctan x}{x}$

3. (9 points) In this problem, we will use linear approximations to estimate the value of $(8.1)^{1/3}$.

   (a) Compute dy/dx for $y = x^{1/3}$, and use this to give the linear approximation $L(x)$ to this function near the point $a = 8$.

$L(x) =$ [ ]

   (b) Using this linear approximation, estimate $(8.12)^{1/3}$. State your answer in exact form.

$(8.12)^{1/3} \approx$ [ ]

4. (10 points) Consider the curve defined implicitly by

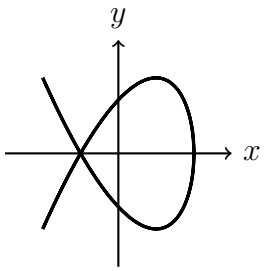
$$xe^y + y\sin(x) - y^2 = -1$$

Find the equation of tangent line to the above curve at the point $(0, 1)$.

5. (12 points) Answer the following questions based on the parametric curve

$$x(t) = \cos(2t) \qquad y(t) = \sin(3t)$$

where $-\frac{\pi}{2} < t < \frac{\pi}{2}$. Its graph is given on the right.

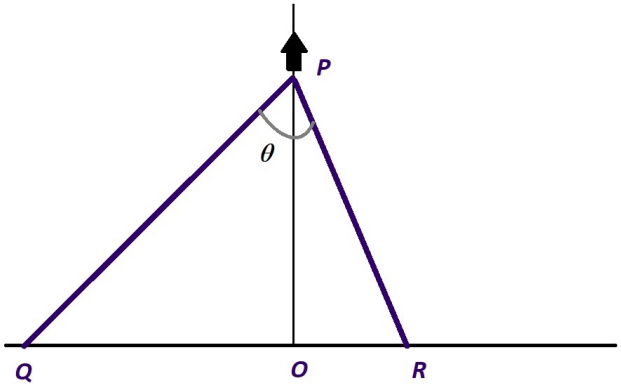


(a) Find the coordinates of all the points on the curve where the tangent line is horizontal.

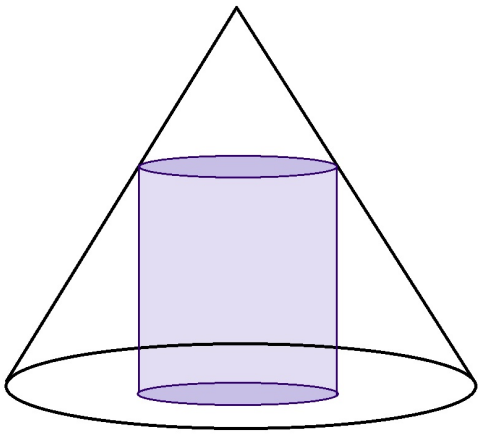
(b) The curve intersects itself at the point (-1/2, 0). Find the equations of the tangent lines at this point. Give your final answers in the form $y = mx + b$.

6. (12 points) There are elastic ropes between points PQ and PR. The distance QO is 8 centimeters and the distance OR is 5 centimeters. Point P is being pulled up so it is rising at a rate of 6 centimeters per second. At what rate is the angle θ at the corner P changing when the distance OP is 10 centimeters?

   Picture is not to scale.



7. (11 points) Find the dimensions of the cylinder of maximum volume contained in a cone of height 6 and radius 8 units. Make sure you verify that your answer indeed gives maximum volume.



8. (19 total points) Let $f(x) = \dfrac{3x^2 - 1}{3x^3}$.

   (a) (2 points) Determine all x and y-intercepts for the curve. If there are none, say so.

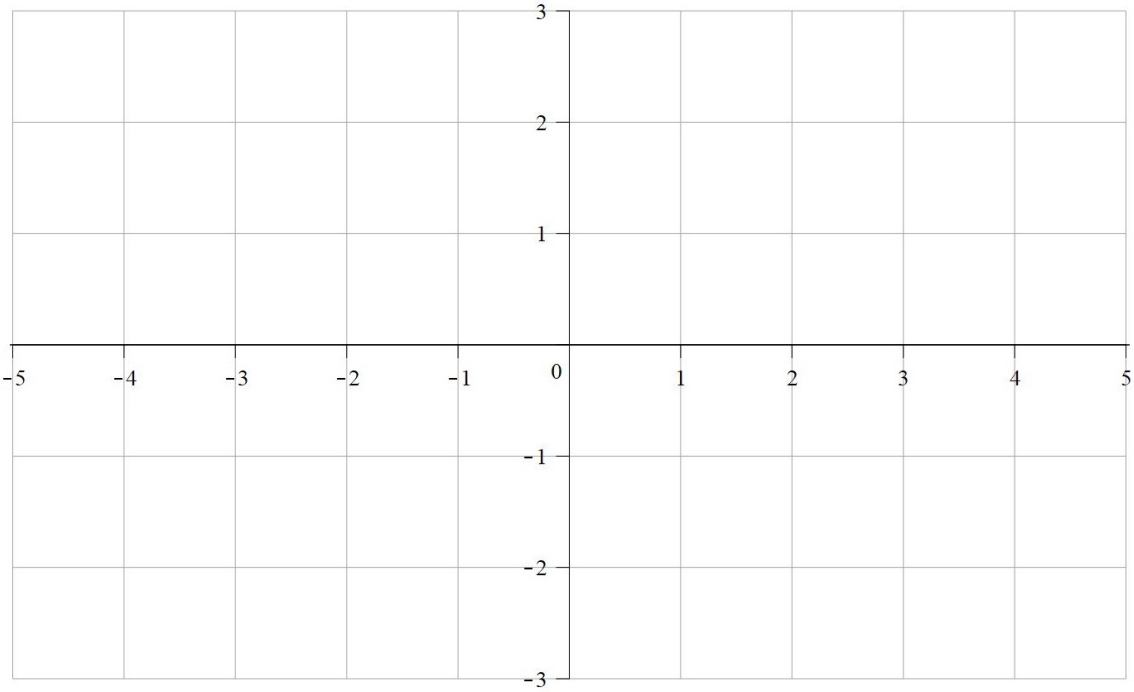
   (b) (3 points) Determine any vertical asymptotes and horizontal asymptotes for the curve $y = f(x)$. If there are none, say so.

   (c) (6 points) Find all critical numbers for $f(x)$. Find the intervals on which $f(x)$ is increasing, and the intervals on which $f(x)$ is decreasing. Determine x and y-coordinates of all local minimum(s) and local maximum(s).

Recall that the function is: $f(x) = \dfrac{3x^2 - 1}{3x^3}$.

(d) (5 points) Find the intervals on which $f(x)$ is concave up and concave down. Find the x and y-coordinates of all of the inflection points.

(e) (3 points) Sketch the curve using ALL of the information in (a)-(d). Mark any important points that came up in your computations.



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