Your Name

Your Signature

Student ID #

Quiz Section

Put a check next to your professor's name

☐   Prof. Charles Camacho

☐   Prof. David Collingwood

☐   Prof. Elena Pezzoli

READ THE INSTRUCTIONS!

- *These exams will be scanned. Write your name, student number and quiz section clearly.*

- Turn off and stow away all cell phones, smart watches, and other similar devices. No earbuds, headphones, or any kind of connected devices allowed during the exam.

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.

- Give your answers in exact form unless instructed otherwise. For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.

- You can only use a Texas Instruments TI-30X IIS calculator.

- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.

- This exam has 11 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

Problem	Score	Problem	Score	Problem	Score
1 (12 pts)		4 (10 pts)		7 (13 pts)	
2 (15 pts)		5 (10 pts)		8 (19 pts)	
3 (9 pts)		6 (12 pts)		**Total**	

1. (12 total points) Find the following limits or explain why the limit does not exist. Give exact answers.

   (a) (4 points) $\displaystyle\lim_{t\to 2}\frac{3^t-9}{t-2}$

   (b) (4 points) $\displaystyle\lim_{x\to\infty}\frac{\sqrt{x^2+x+1}}{\sqrt{7x^2+3}}$

   (c) (4 points) $\displaystyle\lim_{x\to 0}\frac{\sin(5x)}{\tan(3x)}$

2. (15 total points) Compute the first derivatives of the following functions. You do not need to simplify.
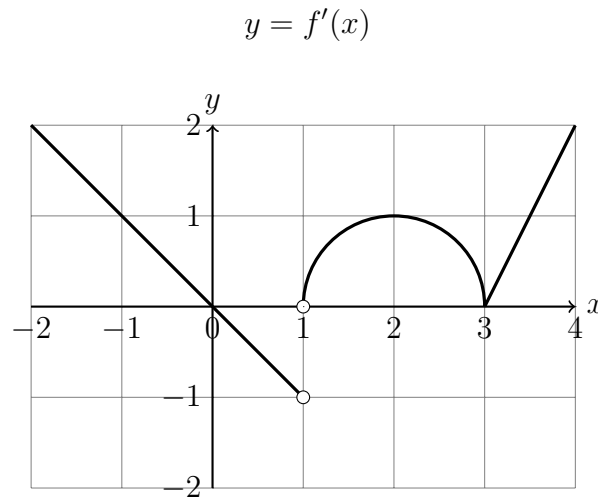
   (a) (5 points) $f(x) = x \arctan(x^2 + 1)$

   (b) (5 points) $g(x) = \dfrac{e^{e^x}}{\ln x}$

   (c) (5 points) $h(x) = x^{\sqrt{x}+1}$

3. (9 points) For this problem, you do not need to show your work. Answer the following questions based on **the graph of the derivative** $y = f'(x)$ below. For questions involving limits, if the limit is infinite, write ∞ or $-\infty$. If the limit does not exist, write DNE. Assume $f(x)$ is continuous on its domain $-2 < x < 4$.

The graph below is a line for $x < 1$; an upper-semicircle for $1 < x < 3$; and a line for $3 < x$.

$$y = f'(x)$$

(a) $f'(2) =$

(b) $f''(0) =$

(c) $\displaystyle \lim_{x \to 0} \frac{f(x) - f(0)}{x} =$

(d) $\displaystyle \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} =$

(e) $\displaystyle \lim_{x \to 3^+} \frac{f'(x)}{x - 3} =$

(f) List the interval(s) where $y = f(x)$ is increasing.

Your answer: _____

(g) List the x-values that are the critical numbers for $y = f(x)$.

Your answer: _____

(h) List the interval(s) where $y = f(x)$ is concave up.

Your answer: _____

(i) List the x-values of the inflection points for $y = f(x)$.

Your answer: _____

4. (10 points) For a certain item you are producing, the price p of the item and the number q of items sold are related by the equation

$$(p - 20)^3 + pq + q^3 = 801.$$

According to this equation, if the price of one item is $p = 5$ dollars, the number of items sold is $q = 16$. Use linear approximation to estimate what the price of one item should be if you want to sell 17 items. Round your answer to the nearest cent.

5. (10 total points) The location of a particle moving in the plane at time t seconds is given by these parametric equations:

$$x = (t-2)^2 \qquad\qquad y = (t-2)^3 - 3(t-2).$$

The path is graphed on the right for $0 \le t \le 4$.
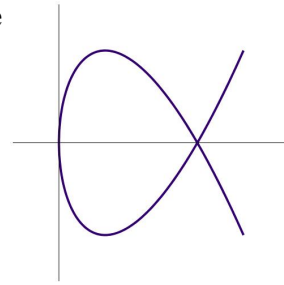
(a) (4 points) Find all of the times when the particle crosses the x-axis.

(b) (4 points) Find the equation of the tangent line to the path the first time the particle crosses the x-axis.

(c) (2 points) Find the equation of the tangent line to the path the last time the particle crosses the x axis.

6. (12 points) An inverted conical tank, having a circular top 4 m in diameter and 6 m tall, is being filled with water at a rate of 5 m³/min (see the figure below). Meanwhile, a hole at the bottom of the tank causes water to leak out of the tank at a rate of 3 m³/min. Find the rate of change of the height of the water in the tank when the water is 3 m high.
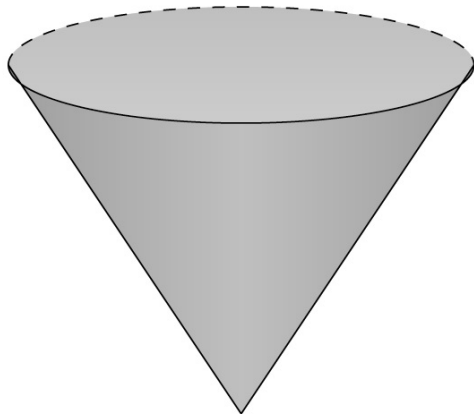
7. (13 points) You want to connect two poles by a wire that is also connected to the ground. The first pole is 20 feet tall and the second pole is 40 feet tall. There is a distance of 60 feet between the two poles. What is the minimum amount of wire needed? Round your answer to one digit after the decimal.

20 ft

40 ft

60 ft
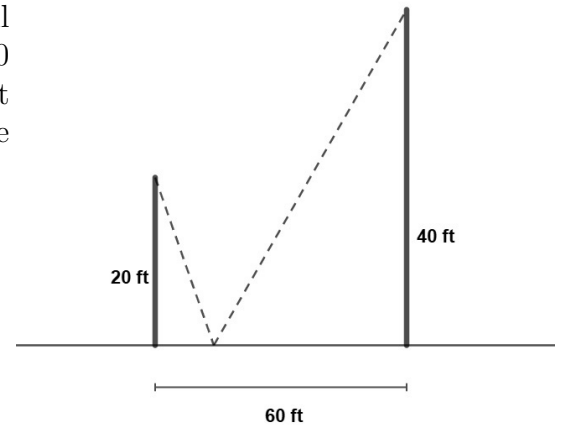
8. (19 total points) Let $y = f(x) = e^x(x-1)^2$ on the domain of all real numbers.

(a) (2 points) Determine all x and y-intercepts for the curve.

(b) (3 points) Determine any vertical asymptotes and horizontal asymptotes for the curve $y = f(x)$.

(c) (6 points) Find all critical numbers for $f(x)$. Find the intervals on which $f(x)$ is increasing, and the intervals on which $f(x)$ is decreasing. Determine x and y-coordinates of all local minimum(s) and local maximum(s).

Recall that the function is: $y = f(x) = e^x(x-1)^2$.

(d) (5 points) Find the intervals on which $f(x)$ is concave up and concave down. Find the x and y-coordinates of all of the inflection. points.

(e) (3 points) Sketch the curve using ALL of the information in (a)-(e). Mark any important points that came up in your computations.
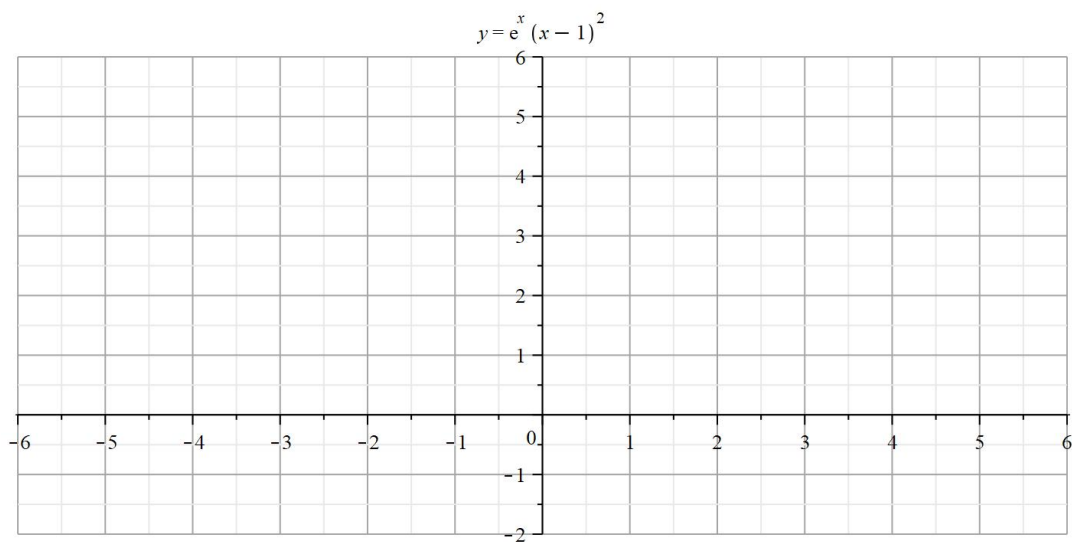
$$y = e^x (x-1)^2$$

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