Your Name
$\square$
Student ID \#
$\square$
Put a check next to your professor's name
$\square$ Prof. Charles Camacho
Prof. David Collingwood
Prof. Fanny Dos Reis

## READ THE INSTRUCTIONS!

- These exams will be scanned. Write your name, student number and quiz section clearly.
- Turn off and stow away all cell phones, smart watches, and other similar devices. No earbuds, headphones, or any kind of connected devices allowed during the exam.
- This exam is closed book. You may use one $8.5^{\prime \prime} \times 11^{\prime \prime}$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- Give your answers in exact form unless instructed otherwise. For example, $\frac{\pi}{3}$ or $5 \sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must show all of your work. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- This exam has 11 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

| Problem | Score | Problem | Score | Problem | Score |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1(12 \mathrm{pts})$ |  | $4(12 \mathrm{pts})$ |  | $7(13 \mathrm{pts})$ |  |
| $2(15 \mathrm{pts})$ |  | $5(12 \mathrm{pts})$ |  | $8(15 \mathrm{pts})$ |  |
| $3(9 \mathrm{pts})$ |  | $6(12 \mathrm{pts})$ |  | Total |  |

1. (12 total points) Compute each of the following limits showing complete work or justification for your answer. If there is no finite limit, write $\infty,-\infty$, or DNE, whichever applies.
(a) (4 points) $\lim _{x \rightarrow 1}\left(\frac{1}{x-1}+\frac{1}{x^{2}-3 x+2}\right)$
(b) (4 points) $\lim _{y \rightarrow 4} \frac{\sqrt{y}}{(y-4)^{6}}$
(c) (4 points) $\lim _{x \rightarrow 1} \frac{\ln x}{\tan (\pi x)}$
2. (15 total points) Find the derivatives of the following functions. Do not simplify your answers.
(a) (5 points) $f(x)=\sin (\sqrt{3 x})+\sqrt{\sin (3 x)}$.
(b) (5 points) $f(x)=\frac{\ln (1+3 x)(\arcsin (7 x))}{e^{2 x}+x}$
(c) (5 points) $g(t)=(\cos (a t))^{\sqrt{b t+c}}$ where $a, b$ and $c$ are constant real numbers.
3. (9 points) For this problem, you do not need to show your work. The function $f(x)$ is continuous on its domain $-4<x<5$.

Answer the following questions based on the GRAPH OF THE DERIVATIVE $y=f^{\prime}(x)$ below. Note that the derivative $f^{\prime}(x)$ is not defined at $x=-4,-1,1,5$. For questions involving limits, if the limit is infinite, write $\infty$ or $-\infty$. If the limit does not exist, write DNE. You may have to give an approximate a value as an answer and this will be considered while grading.

The graph of the derivative below is a line between $-4 \leq x<-1$; the upper semi circle of radius 1 centered at $(0,3)$ for $-1<x<1$; and a parabola between $1<x<5$.

(a) $\lim _{x \rightarrow-1} f^{\prime}(x)=$
(b) $f^{\prime}(0)=$
(c) $\lim _{x \rightarrow 1^{-}} f^{\prime \prime}(x)=$
(d) $\lim _{h \rightarrow 0} \frac{f(h)-f(0)}{h}=$
(e) $\lim _{x \rightarrow 3} \frac{f^{\prime}(x)+2}{x-3}=$
(f) Let $g(x)=f\left(f^{\prime}(x)\right)$.

Then $g^{\prime}(-2)=$
(g) List the $x$-value(s) for $-4<x<5$ corresponding to the inflection point(s) for $y=f(x)$.

Your answer: $\qquad$
(h) List the $x$-value(s) for $-4<x<5$ where $y=f(x)$ attains a local maximum.

Your answer: $\qquad$
(i) List the $x$-value(s) for $-4<x<5$ where $y=f(x)$ attains a local minimum.

Your answer: $\qquad$
4. (12 total points) Given the curve

$$
x^{2 / 3}+y^{2 / 3}=5
$$

answer the following.
(a) (6 points) Verify that the point $(8,1)$ is on the curve and find the equation of the tangent line to the curve at this point.
(b) (6 points) Is the graph concave up or concave down at that point?
5. (12 total points) Given the parametric curve

$$
x(t)=t^{3}-t+1, \quad y(t)=t^{2}+1
$$

answer the following questions.
A graph is given on the right to help you visualize.

(a) (4 points) Find the coordinates of the point where the curve crosses itself, where two different $t$ values give the same point $(x, y)$.
(b) (4 points) Find equations of the 2 tangent lines at the point where the curve crosses itself.
(c) (4 points) Find the coordinates of the points where the tangent lines are vertical.
6. (12 points) A pool is 10 meters long, 5 meters wide, 1 meter deep at the shallow end and 4 meters deep at the deep end.
A cross section is shown below:


The water is drained out of the pool for cleaning at a rate of 0.25 cubic meters per minute. How fast is the water level changing when the depth of the water at the deepest point is 2 meters?
7. (13 points) Jordan has a backyard with an elliptical pond measuring 6 feet wide horizontally and 4 feet vertically. Find the area of the largest rectangular platform that Jordan can place inside the pond whose corners touch the pond's boundary.
Hint: Start with writing down an equation of the ellipse centered at the origin.

8. (15 total points) This problem will work with the function $f(x)=\frac{x^{2}+x+1}{x^{2}}$ on the domain of all non-zero real numbers.
(a) (2 points) Determine any vertical asymptotes for the curve (show your limit computations).
(b) (2 points) Determine any horizontal asymptotes for the curve (show your limit computations).
(c) (4 points) Find all critical numbers for $f(x)$.

Recall that the function is: $f(x)=\frac{x^{2}+x+1}{x^{2}}$.
(d) (3 points) Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. Determine $x$ and $y$ coordinates of all local minimum(s) and local maximums(s).
(e) (4 points) Find the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down. Find the $x$ and $y$ coordinates of all of the inflection points.

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