Math 124 **Final Examination** **Winter 2023**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

READ THE INSTRUCTIONS!

- *These exams will be scanned. Write your name, student number and quiz section clearly.*
- Turn off and stow away all cell phones, smart watches, mp3 players, and other similar devices. No earbuds/headphones allowed during the exam.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- Give your answers in exact form unless instructed otherwise. For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- This exam has 11 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

Problem	Score	Problem	Score	Problem	Score
1 (12 pts)		4 (12 pts)		7 (12 pts)	
2 (12 pts)		5 (12 pts)		8 (16 pts)	
3 (12 pts)		6 (12 pts)		**Total**	

1. (12 total points) Compute each of the following limits. If there is no finite limit, write ∞, $-\infty$, or DNE (does not exist), whichever applies.

   (a) (4 points) $\lim_{x\to 0} \dfrac{x \sin x}{1 - \cos x}$

   (b) (4 points) $\lim_{x\to 1} \dfrac{x^2 - 3x + 2}{x - 1}$

   (c) (4 points) $\lim_{x\to \infty} \cos\left(\dfrac{\ln(\ln x)}{\ln x}\right)$

2. (12 total points) Find the derivatives of the following functions. You do not have to simplify.

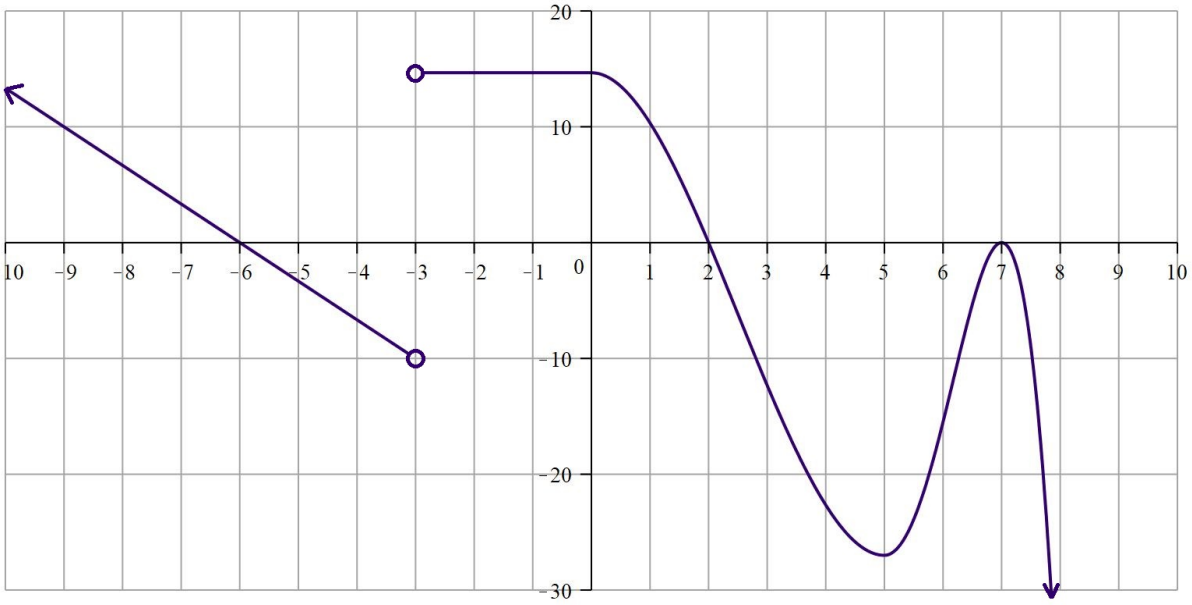
   (a) (4 points) $y = (1 + \sqrt{t})^{20}(1 + t)^{23}$.

   (b) (4 points) $y = (\sin x + \tan^3 x)^{\frac{1}{3}}$.

   (c) (4 points) $y = (1 + \cos x + e^x)^{\sin x}$.

3. (12 points) The function $f(x)$ is differentiable everywhere except at $x = -3$. Answer the questions based on the graph of $y = f'(x)$, **the derivative of** $f(x)$, shown below.

$y = f'(x)$



(a) $\lim_{x \to -3^+} f'(x) =$

(b) $f''(-7) =$

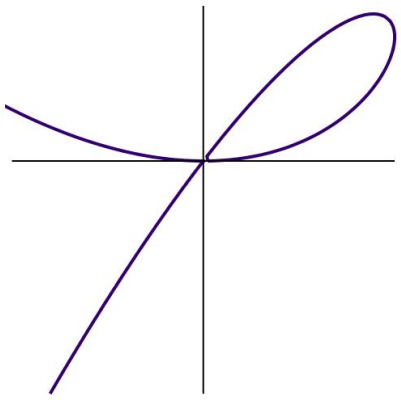
(c) List all values of x where the graph of $y = f(x)$ has a local maximum.

(d) List all intervals where the graph of $y = f(x)$ is concave up.

(e) List all x values where the graph of $y = f(x)$ has a point of inflection.

(f) If $f(-2) = 0$, what is $f(-0.5)$?

4. (12 points) Consider the curve defined by the implicit function $x^3 - 4xy + y^2 = 0$ whose graph is shown on the right.



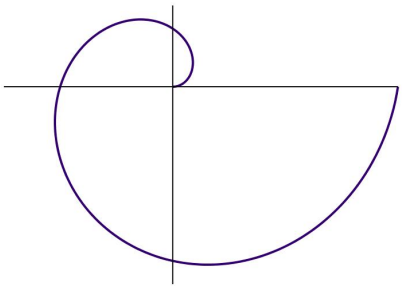
   (a) Find all points (a, b) on the curve where the tangent line is vertical.

   (b) Check that the point $(3, 3)$ lies on the curve. Use a linear approximation to estimate the x-coordinate of a point on the curve with y-coordinate equal to 2.95.

5. (12 points) A particle is moving in the plane and has parametric equations

$$x(t) = t\cos(\pi t) \qquad y(t) = t\sin(\pi t)$$

where $t \geq 0$. The path of the particle during the time interval $0 \leq t \leq 2$ is plotted on the right.



(a) What is the horizontal velocity of the particle at time t?

(b) What is the vertical velocity of the particle at time t?

(c) What is the equation of the tangent line to the path when the particle crosses the negative y-axis?

(d) If $s(t)$ is the speed of the particle at time t,

$$\lim_{t\to\infty} s(t) =$$

6. (12 points) The top of a ladder slides down a vertical wall at a rate of 0.25 meters per second. At the moment when the bottom of the ladder is 5 meters from the wall, it slides away from the wall at a rate of 0.6 meters per second. How long is the ladder?

7. (12 points) A lump of clay of volume 1000 cubic centimeters is used to make a cube and a sphere. Find the dimensions of the cube and the sphere that would give the maximum and the minimum total surface areas.
   Recall that the volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$ and its surface area is given by $A = 4\pi r^2$

8. (16 points) Let $f(x)$ be the function

$$y = f(x) = 2 - \frac{6}{x} + \frac{6}{x^2}$$

on the domain of all non-zero real numbers.

(a) Find all intervals over which $f(x)$ is decreasing.

(b) Find all intervals over which $f(x)$ is concave down.

Continued on the next page.

8. (continued) Recall the function $y = f(x) = 2 - \dfrac{6}{x} + \dfrac{6}{x^2}$

(c) Calculate the following limits.

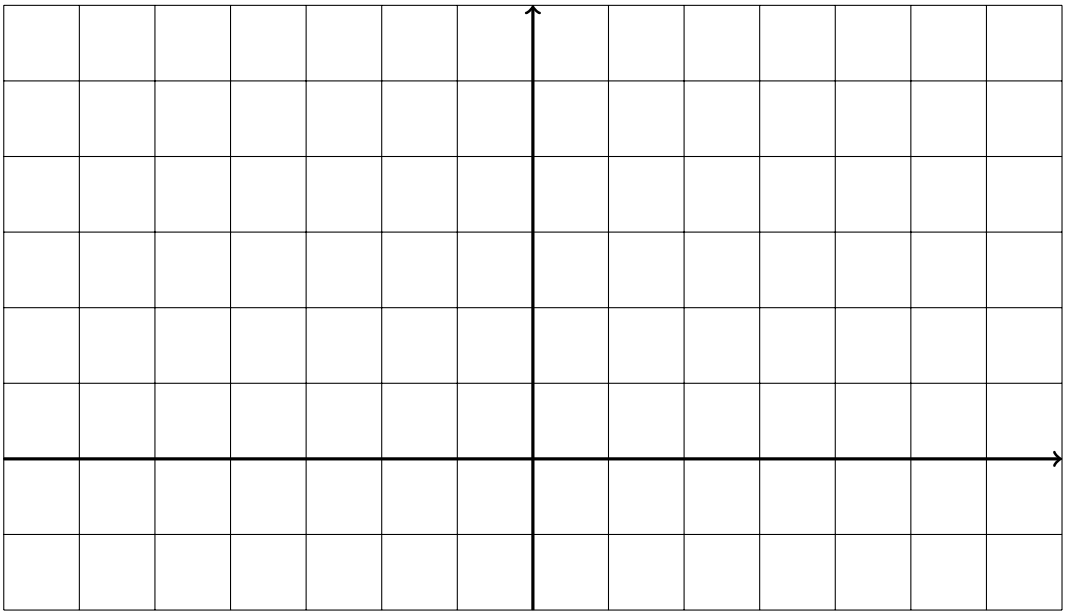
(i) $\lim\limits_{x\to\infty} f(x)$

(ii) $\lim\limits_{x\to-\infty} f(x)$

(iii) $\lim\limits_{x\to0^+} f(x)$

(iv) $\lim\limits_{x\to0^-} f(x)$

(d) Sketch the graph $f(x)$ using the grid below. Clearly label the (x, y) coordinates of all critical points and all points of inflection.



This page is blank. If you continued a question here, make a note on the question page so we check it.

This page is blank. If you continued a question here, make a note on the question page so we check it.